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Abstract

Many countries are adopting austerity measures, whereby governments aggressively raise taxes, with the hope to dispel future solvency crisis. This paper investigates the implications of austerity on the likelihood of solvency crisis. We derive the maximum level of debt consistent with solvency, labelled as the effective fiscal limit on debt, and we show that its position depends on austerity. We find that countries like Italy that undergo strict austerity could lower their effective fiscal limit and induce a solvency crisis in the near future.

- *Key Words:* Austerity, Solvency Crisis, Fiscal policy, Fiscal limit, Default
- *JEL Classification*: C63, E62, E63, F34, H63

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1 Introduction

The solvency crisis in Europe sparked a new era of austerity, whereby governments must aggressively raise taxes. Country after country are pressured to adopt austerity measures to rein in their elevated debt levels with the hope to allay the possibility of future solvency crisis. But, do austerity measures avert the likelihood of future insolvency? In Greece, after seven years of strict austerity measures demanded by international creditors, there are no signs that the government can escape the crisis and reattain a solvent position in the near future. In Italy, the government is resisting additional austerity and is considering easing up on austerity, despite its ever-growing debt level. The EU officials appear to believe that austerity measures avert solvency crisis, whereas some European countries appear to doubt whether austerity will ever end a crisis or prevent one. Here, we evaluate the efficacy of austerity.

We use a small open economy model in a monetary union in which the government collects distortionary taxes, finances transfers and government purchases, and issues bonds. The government increases the taxes to reduce debt. Distortionary taxes limit the tax revenue that the government can generate, which in turn allows us to derive the maximum level of debt that the government can repay, which we call the "effective fiscal limit." Beyond this maximum value, debt embarks on an explosive path, creditors flee, and the country faces a solvency crisis. To prevent a solvency crisis, debt needs to remain below this effective fiscal limit. We define austerity as the aggressive increase in taxes to retire debt,[1] and investigate

[1] Austerity is also referred to as fiscal consolidation, or fiscal contraction, or more simply as an increase in taxes.

the implications of austerity on the effective fiscal limit. Austerity measures that raise taxes to the peak of the Laffer curve - the point that maximizes tax revenue - could raise the effective fiscal limit. Austerity measures, however, that push the taxes to the slippery side of the Laffer curve lower the effective fiscal limit and raise the likelihood of future insolvency. The first takeaway of our analysis is that austerity affects the position of the effective fiscal limit. The second takeaway is that there is a nonlinear relationship between austerity and the effective fiscal limit, and equivalently between austerity and solvency crisis.

Most of the austerity literature focuses on disentangling the effects of austerity on output. In traditional Keynesian models, austerity contracts aggregate demand and reduces output. In contrast, Giavazzi and Pagano (1990) suggest that austerity could be expansionary. The impact of austerity, however, goes beyond output. In this paper, we show that austerity affects the maximum level of debt consistent with solvency.

The literature offers two concepts for the maximum level of debt. Bi (2012) and Bi et al. (2013) derive their fiscal limit on debt from the top of the Laffer curve. Combining the peak of the Laffer curve with the government's intertemporal budget constraint, they obtain their fiscal limit on debt. Ghosh et al. (2013) identify their debt limit using a cubic reduced-form regression of the primary surplus on debt. A negative coefficient on the cubic term yields an unstable region. Once debt enters the unstable region, debt embarks on an explosive path. The value of debt on the boundary of the unstable region represents their debt limit.

We draw on these models and propose an alternative procedure to derive the maximum level of debt consistent with solvency. Our approach combines the dynamic Laffer curves of Bi (2012) with the unstable regions of Ghosh et al. (2013). The effective fiscal limit presented in this paper can be viewed as adding distortionary taxes in the Ghosh et al. (2013) model.

By endogenizing the output growth rate, the model naturally yields unstable regions, and it does not require Ghosh et al.'s (2013) assumptions of a nonlinear fiscal feedback function with a negative coefficient on the cubic term and low interest rates to obtain an estimate of the effective fiscal limit. Or alternatively, this paper can be viewed as extending Bi's (2012) model in two key ways. First, we introduce persistence in the tax feedback rule, capturing the inertia in the legislative and implementation process as in Daniel and Shiamptanis (2012, 2013), and we find that the level of the tax rate affects the effective fiscal limit. We obtain a hump-shaped effective fiscal limit where medium tax rates yield higher maximum values of debt consistent with solvency, than low and high tax rates, implying that countries with either very low and very high tax rates can experience a solvency crisis at a lower level of debt. Second, following Daniel and Shiamptanis (2017) we relax the assumption that the peak of the Laffer curve can be attained instantaneously and maintained forever, and we find that all the fiscal policy parameters, including the tax adjustment parameter which is our proxy for austerity, affect the position of the effective fiscal limit.[2] By mapping all the fiscal policy parameters into the effective fiscal limit, this paper provides a tool to investigate the implications of austerity on solvency crisis.

Our paper is also related to Arellano and Bai (2016) who study the linkage between austerity and solvency crisis. Our model is analogous to their fiscal default. In their paper, however, they find that in the presence of fiscal constraints, austerity lowers the likelihood of a solvency crisis. In contrast, we find a nonlinear relationship between austerity and the probability of a solvency crisis. For very small values of the tax adjustment parameter,

[2] Trabandt and Uhlig (2011) find that none of the countries in their sample are at the peak of the Laffer curve, but rather they are on either side of the Laffer curve.

austerity is successful in lowering the crisis risk, whereas for other values, austerity raises the crisis risk.

The final contribution of the paper is quantitative. We apply our model to Italy. We estimate the Italian effective fiscal limit and quantify the probability of solvency crisis. Next we use our model to ask whether austerity measures could alter solvency risk in Italy. We find that if Italy adopts austerity to lower its debt, it will also lower its effective fiscal limit, thereby raising the danger of future insolvency.

This paper is organized as follows. Section 2 presents the model, derives the effective fiscal limit and maps austerity to the effective fiscal limit. Section 3 applies the model to Italy and Section 4 provides conclusions.

2 Model

We set up a simple small open economy model in a monetary union. The country is small enough that it cannot affect the foreign interest rate. The country faces an effective fiscal limit, which arises endogenously from Laffer curves. Solvency requires that debt remains below the effective fiscal limit. In the event of a solvency crisis, the country partially defaults. We assume that a solvent government always repay.[3]

2.1 Household

The small open economy is populated by a representative household, who chooses consumption (c_t) and leisure (l_t), and maximizes the following utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

[3] Default is due to the inability to repay, and not due to a strategic decision as in Arellano (2008)

subject to the following budget constraint

$$b_t^d = (1 + i_{t-1})\, \delta_t b_{t-1}^d + (1 - \tau_t)\, y_t + z_t - c_t$$

where b_t^d denotes the bond purchases, i_{t-1} is the interest rate that the bond pays, δ_t is the fraction of the value of the bonds that will be repaid at time t,[4] y_t is output, τ_t represents the distortionary labor income tax rate, z_t is the lump-sum transfers to the household by the government, and $0 < \beta < 1$ is the discount factor. E_t denotes the expectation conditional on the information at time t.

Output is determined by the productivity level (A_t) and the labor supply $(1 - l_t)$,

$$y_t = A_t\,(1 - l_t)$$

where the productivity level follows an AR(1) process with A representing the steady-state level and ε_t^A the productivity shocks

$$A_t - A = \rho^A\,(A_{t-1} - A) + \varepsilon_t^A, \qquad \varepsilon_t^A \sim N\left(0, \sigma^A\right).$$

The household's maximization problem yields the typical first-order conditions

$$\begin{aligned} \beta E_t \left(\frac{U_{c_{t+1}}}{U_{c_t}} \delta_{t+1} \right) &= \frac{1}{(1 + i_t)}, \\ \frac{U_{l_t}}{U_{c_t}} &= (1 - \tau_t)\, A_t. \end{aligned}$$

2.2 Government

We assume that the government issues bonds (b_t) in the common currency, which are either held by the domestic agent (b_t^d) or the foreign agent (b_t^f), such that $b_t = b_t^d + b_t^f$. The government's real flow budget constraint is given by

$$b_t = (1 + i_{t-1})\, \delta_t b_{t-1} - \tau_t y_t + g_t + z_t.$$

[4] $\delta_t = 1$ implies no default, where $\delta_t < 1$ implies partial default.

where $\tau_t y_t$ is tax revenue, g_t is government purchases, and z_t is transfer payments. In the event of a solvency crisis, the government repays a fraction of its outstanding liabilities, δ_t.

We assume that the foreign agent is willing to buy the government bonds (b_t^f) as long as the domestic interest rate (i_{t-1}) satisfies interest rate parity. Interest rate parity is derived from the foreign agent's Euler equations when the covariance between the domestic interest rate and the foreign agent's consumption is zero[5], and it can be expressed as

$$1 + i = (1 + i_{t-1})\, E_{t-1}\delta_t \tag{1}$$

where i is the foreign default-free interest rate, which is assumed to be constant. Equation (1) implies that the domestic country's interest rate (i_{t-1}) rises above the foreign interest rate (i) when there is some possibility of default ($E_{t-1}\delta_t < 1$).

Imposing interest rate parity from equation (1) and rearranging yields

$$b_t = (1 + i)\, b_{t-1} - \tau_t y_t + g_t + z_t - (E_{t-1}\delta_t - \delta_t)\,(1 + i_{t-1})\, b_{t-1}.$$

Define the capital loss on debt due to default, denoted by a_t, as

$$a_t = (1 - \delta_t)\,(1 + i_{t-1})\, b_{t-1} \tag{2}$$

where default ($\delta_t < 1$) increases the capital loss on debt ($a_t > 0$). Using equation (2), the evolution of government's debt can be expressed as

$$b_t = (1 + i)\, b_{t-1} - \tau_t y_t + g_t + z_t - (a_t - E_{t-1}a_t) \tag{3}$$

and $a_t - E_{t-1}a_t$ represents the unexpected default, which reduces the value of debt and contributes to government revenue.

[5] This follows from the small open economy assumption. This also holds when the foreign agent is risk neutral.

2.3 Fiscal policy feedback rules

We assume that the fiscal policy adjusts the tax rate (τ_t) in response to increases in debt

$$\tau_t - \tau = \rho^{\tau} (\tau_{t-1} - \tau) + \gamma (b_{t-1} - b) \tag{4}$$

where τ and b are the steady state values of the tax rate and debt, respectively, ρ^{τ} measures the persistence in the tax rate, which captures the inertia in the legislative and implementation process, and γ is the tax adjustment parameter, which captures the responsiveness of the tax rate to increases in debt and is our measure for austerity. We refer to an increase in γ as "austerity." A stronger responsiveness to debt implies that the government is raising the tax rate aggressively to retire debt.

Government purchases and transfers are specified as AR(1) processes[6]

$$\begin{aligned} g_t - g &= \rho^g (g_{t-1} - g) + \varepsilon_t^g, \quad \varepsilon_t^g \sim N(0, \sigma^g), \\ z_t - z &= \rho^z (z_{t-1} - z) + \varepsilon_t^z, \quad \varepsilon_t^z \sim N(0, \sigma^z) \end{aligned}$$

where ρ^g and ρ^z measure the persistence in government spending and transfers, g and z represent the value of government purchases and transfers at the steady state, and ε_t^g and ε_t^z represent the government spending and transfer shocks, respectively. The shocks are random and represent both unanticipated shocks, as well as discretionary responses to the state of the economy.

2.4 Dynamics

It is useful to represent the dynamic behavior of the tax rate and debt system using a phase diagram, which reveals the direction of movement of the tax rate and debt at each point.

[6] The AR(1) specifications capture the Italian behavior between 1970 and 2015. We find that Italian fiscal authorities do not cut government spending and transfers when debt rises.

We construct the phase diagram of the system by subtracting the lagged value of the tax rate from equation (4), the lagged value of debt from equation (3), and setting them equal to zero to yield

$$\Delta\tau_t = \tau_t - \tau_{t-1} = (\rho^\tau - 1)(\tau_{t-1} - \tau) + \gamma(b_{t-1} - b) = 0 \tag{5}$$

$$\Delta b_t = b_t - b_{t-1} = ib_{t-1} - \tau_t y_t + g_t + z_t - (a_t - E_{t-1}a_t) = 0 \tag{6}$$

We assume that the household's utility function is given by $U(c_t, l_t) = \log c_t + \phi \log l_t$, where ϕ is a preference leisure parameter. Using the household's first-order conditions, output can be written as

$$y_t = \frac{A_t(1-\tau_t) + \phi(1-\lambda)g_t - \phi\lambda z_t}{1 + \phi - (1+\phi\lambda)\tau_t} \tag{7}$$

where $\lambda = \frac{b_t^f}{b_t}$ denotes the fraction of the government's bonds held by the foreign agent.

Setting the shocks equal to their expected values of zero $\left(\varepsilon_t^A = \varepsilon_t^g = \varepsilon_t^z = a_t = E_{t-1}a_t = 0\right)$ and $g_{t-1} = g$, $A_{t-1} = A$, $z_{t-1} = z$, the two equations for the phase diagram are given by

$$b_{t-1}|_{\Delta\tau_t=0} = \frac{\gamma b + (\rho^\tau - 1)\tau + (1-\rho^\tau)\tau_t}{\gamma} \quad \text{and} \tag{8}$$

$$b_{t-1}|_{\Delta b_t=0} = \frac{\tau_t\left[A(1-\tau_t) + \phi(1-\lambda)g - \phi\lambda z\right]}{\left[1+\phi-(1+\phi\lambda)\tau_t\right]i} - \frac{g+z}{i}. \tag{9}$$

The phase diagram is presented in Figure 1. Debt (b_{t-1}) is on the vertical axis and the tax rate (τ_t) is on the horizontal axis. The $\Delta\tau = 0$ curve, equation (8), is linear and it has a positive slope, $\frac{1-\rho^\tau}{\gamma} > 0$. The $\Delta b = 0$ curve, equation (9), is nonlinear and its shape mimics the shape of the Laffer curve. The $\Delta\tau = 0$ and $\Delta b = 0$ curves intersect at points G and H. Only point G is stable and it represents the long-run equilibrium in which the tax rate and debt are equal to their steady state values $\left(\tau^G = \tau,\ b^G = b\right)$. At point H, the values of the

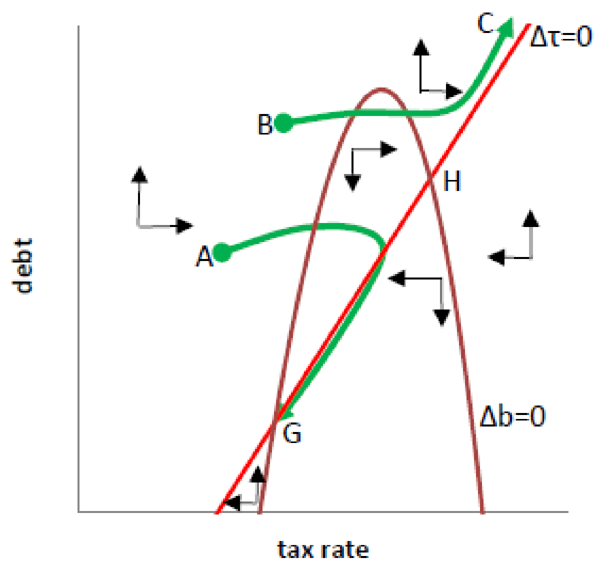

Figure 1: Phase diagram

tax rate and debt are given respectively by

$$\tau^H = 1 - \tau + \frac{((1-\rho^\tau)\,\tau - \gamma b)\, i\,(1+\phi\lambda) - g\gamma\,(1+\phi) - z\gamma + (1-\rho^\tau)\, i\phi\,(1-\lambda)}{(1-\rho^\tau)\, i\,(1+\phi\lambda) - \gamma A},$$

$$b^H = b + \frac{(\rho^\tau - 1)\,\tau + (1-\rho^\tau)\,\tau^H}{\gamma}.$$

Even if the initial tax rate and debt are at point H, the system is expected to travel towards point G, eventually reaching its long-run equilibrium. However, if the system is in the region north of point H, the system fails to attain its long-run equilibrium. In this region, the debt embarks on an explosive path and is therefore inconsistent with solvency. This is a locally stable model, implying that the system is expected to reach its long-run equilibrium for only some values of the tax rate and debt. The adjustment path AG reflects a stable path of the tax rate and debt towards point G, whereas the adjustment path BC represents an explosive path.

Dividing equation (6) by (5) yields the time-varying slope of any adjustment path as

$$\frac{\Delta b_t}{\Delta s_t} = \frac{ib_{t-1} + g + z - \left[(1-\rho^\tau)\tau - \gamma b + \rho^\tau \tau_{t-1} + \gamma b_{t-1}\right]\left[\frac{A(1-(1-\rho^\tau)\tau + \gamma b - \rho^\tau \tau_{t-1} - \gamma b_{t-1}) + \phi(1-\lambda)g - \phi\lambda z}{1+\phi-(1+\phi\lambda)((1-\rho^\tau)\tau - \gamma b + \rho^\tau \tau_{t-1} + \gamma b_{t-1})}\right]}{(\rho^\tau - 1)(\tau_{t-1} - \tau) + \gamma(b_{t-1} - b)}, \tag{10}$$

which can be positive or negative depending on the values of the tax rate and debt.

2.5 Effective Fiscal limit

We exploit the unstable regions in our model created by the dynamic Laffer curves to derive the maximum values of debt consistent with solvency, which we label as the "effective fiscal limit." The boundary of the unstable region represents our measure for the effective fiscal limit. Figure 2 shows this critical boundary DEH. Beginning at any position below DEH, the economy is expected to reach its long-run equilibrium, point G. If debt were to ever breach DEH, the primary surplus would be less than the interest payments and the system would embark on an explosive path. Agents would refuse to lend, creating a solvency crisis. Therefore, in equilibrium the system should remain below DEH.

We approximate the value for debt along the effective fiscal limit, which we label $\hat{b}_t$, by using its previous period value, $\hat{b}_{t-1}$, together with the slope of the adjustment path, given by equation (10), and the change in the tax rate $(\tau_t - \tau_{t-1})$, given by equation (5), to yield

$$\hat{b}_t = \hat{b}_{t-1} + \hat{\zeta}_{t-1}(\tau_t - \tau_{t-1}), \tag{11}$$

where $\hat{\zeta}_{t-1}$ is the slope of the effective fiscal limit DEH and is given by

$$\hat{\zeta}_{t-1} = \frac{i\hat{b}_{t-1} + g + z - \left[(1-\rho^\tau)\tau - \gamma b + \rho^\tau \tau_{t-1} + \gamma \hat{b}_{t-1}\right]\left[\frac{A\left(1-(1-\rho^\tau)\tau + \gamma b - \rho^\tau \tau_{t-1} - \gamma \hat{b}_{t-1}\right) + \phi(1-\lambda)g - \phi\lambda z}{1+\phi-(1+\phi\lambda)\left((1-\rho^\tau)\tau - \gamma b + \rho^\tau \tau_{t-1} + \gamma \hat{b}_{t-1}\right)}\right]}{(\rho^\tau - 1)(\tau_{t-1} - \tau) + \gamma\left(\hat{b}_{t-1} - b\right)}.$$

The slope is positive, $\hat{\zeta}_{t-1} > 0$, when the tax rate and debt are rising along DE, $\hat{\zeta}_{t-1} = 0$ once the effective fiscal limit reaches its peak at point E, and $\hat{\zeta}_{t-1} < 0$ along EH. Our effective

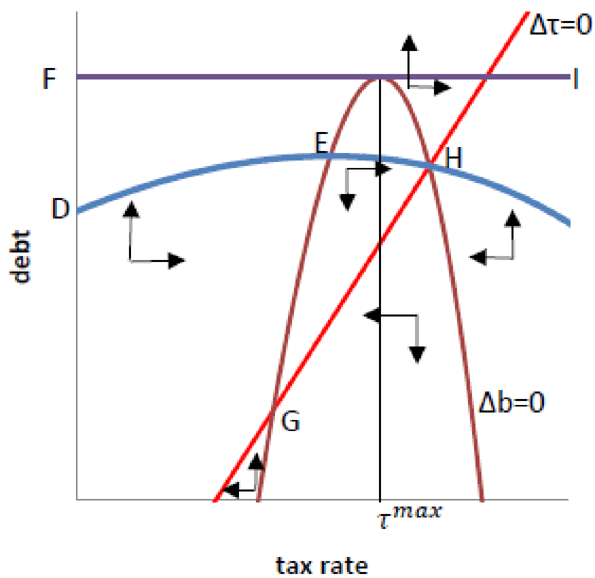

Figure 2: Effective fiscal limit

fiscal limit is nonlinear, with values depending on the level of the tax rate. For low values of the tax rate, the effective fiscal limit is upward-sloping until it peaks at the point which it first intersects with the $\Delta b = 0$ curve (point E). For larger values of the tax rate beyond point E, the critical boundary is downward-sloping. Our hump-shaped effective fiscal limit implies that when the tax rate is either too low or too high, a country could experience a solvency crisis at lower levels of debt.[7]

2.6 Contacts with the literature

The literature proposes two concepts for the maximum level on debt. The fiscal limit in Bi (2012) is derived from the peak of the Laffer curve. Since taxes are distortionary, there is a maximum level of tax revenue that the government can raise. Using equation (7), the tax

[7] Persistence in the tax rate (ρ^{τ}) is critical for the hump-shaped specification. The effective fiscal limit becomes horizontal and goes through point H as ρ^{τ} approaches zero $\left(\lim_{\rho^{\tau}\to 0} \hat{\zeta}_{t-1} \to 0\right)$.

rate that maximizes tax revenue, denote by $\tau_t^{\max}$, can be written as

$$\tau_t^{\max} = \frac{2(1+\phi)A_t - \sqrt{4(1+\phi)^2 A_t^2 - 4(A_t + \phi(1-\lambda)g_t - \phi\lambda z_t)(1+\phi)(1+\phi\lambda)A_t}}{2(1+\phi\lambda)A_t}. \tag{12}$$

Bi's (2012) fiscal limit, which we label as $\hat{b}_t^{Bi}$, is the expected present value of the future maximum primary surpluses that the government can raise

$$\hat{b}_t^{Bi} = E_t \sum_{k=1}^{\infty} \left(\tau_{t+k}^{\max} y_{t+k} - g_{t+k} - z_{t+k}\right) \left(\prod_{j=1}^{k} \frac{1}{1+i_{t+j}}\right). \tag{13}$$

Since shocks affect the future maximum primary surpluses, Bi (2012) derives a distribution for the fiscal limit, not just a point.

In Bi's (2012) procedure, however, the fiscal limit does not depend on all the fiscal policy variables. First, the tax rate (τ_t) does not affect equations (12) and (13), implying that a country with a low tax rate has the same fiscal limit as a country with a high tax rate. Put differently, the fiscal limit is the same, regardless of the tax rate being at its steady state value or at different level. In Figure 2, Bi's (2012) fiscal limit is represented by the horizontal line FI, which is tangent to the peak of the $\Delta b = 0$ schedule.[8] Second, the tax adjustment parameter (γ) does not affect the tax rate at the peak of the Laffer curve, equation (12), and the fiscal limit, equation (13), suggesting that a country that undergoes austerity has the same fiscal limit as a country that eases on austerity.

The Ghosh et al. (2013) approach is based on the stability properties of a fiscal reaction function that governs the evolution of the primary surplus ($s_t = \tau_t y_t - g_t - z_t$). They use a

[8] It is important to note that the phase diagram illustrates both Bi's fiscal limit and this paper's effective fiscal limit when shocks are equal to their expected values of zero. Bi (2012) has shown that shocks yield a distribution for her fiscal limit along the horizontal line FI. When we allow the shocks to take nonzero values, our effective fiscal limit also has a distribution around the DEH curve.

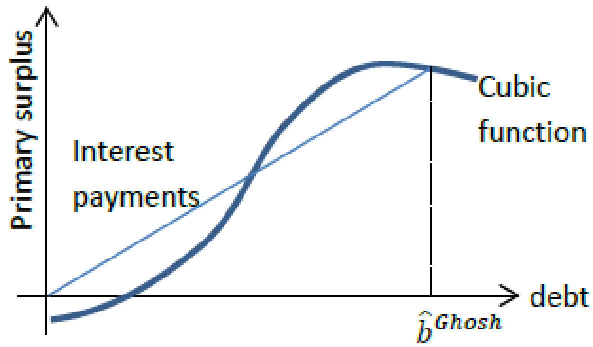

Figure 3: Debt limit of Ghosh et al. (2013)

cubic reduced-form regression of the primary surplus on debt, given by

$$s_t = c + \gamma_1 b_{t-1} + \gamma_2 b_{t-1}^2 + \gamma_3 b_{t-1}^3 + \varepsilon_t^s,$$

where ε_t^s represents the primary surplus shocks. A negative coefficient on the cubic debt term ($\gamma_3 < 0$) implies that the primary surplus weakens as debt increases, a phenomenon that the authors termed as "fiscal fatigue," and eventually there is a point beyond which the primary surplus is not sufficient to pay the interest on debt. At that point, the dynamic system becomes unstable. When debt enters this unstable region, it becomes explosive and agents refuse to lend, creating a solvency crisis. Ghosh et al. (2013) argue that the point at the boundary of the unstable region represents the debt limit, which we label as $\hat{b}^{Ghosh}$, and is given by the largest root of the following equation

$$c + \gamma_1 b_{t-1} + \gamma_2 b_{t-1}^2 + \gamma_3 b_{t-1}^3 + \varepsilon_t^s = i b_{t-1} \tag{14}$$

which equates the cubic function of the primary surplus with the growth-adjusted interest payments (ib_{t-1}), and is illustrated in Figure 3.

The Ghosh et al. (2013) procedure, however, relies on two crucial assumptions. First, it requires a nonlinear fiscal reaction function with $\gamma_3 < 0$. If either γ_3 is positive or the fiscal reaction function is linear, then the Ghosh et al. (2013) approach cannot identify a debt limit. Second, the estimation of the debt limit in equation (14) requires a somewhat low growth-adjusted interest rate (i). A very high i rotates the growth-adjusted interest payment curve counterclockwise around the origin such that equation (14) has no solution.

Our effective fiscal limit, given by equation (11), extends both papers as it maps all the fiscal policy parameters to the fiscal limit and it does not require the assumptions of Ghosh et al. (2013).

2.7 Implications of austerity

Next, we consider the implications of austerity on our effective fiscal limit. We represent austerity with an increase in the value of γ. We find that increases in γ could either raise or lower the effective fiscal limit, depending on the position of the unstable region, which is determined by the unstable point H.

Consider the case where the initial γ is large enough such that $\tau^H > \tau_t^{\max}$. In Figure 4, a further increase in γ reduces the slope of the $\Delta\tau = 0$ curve and rotates it clockwise around point G, moving point H to the right $\left(\frac{\partial \tau^H}{\partial \gamma} > 0\right)$ and downwards $\left(\frac{\partial b^H}{\partial \gamma} < 0\right)$, and thus increasing the area that the dynamics become explosive. Therefore, an increase in γ lowers the effective fiscal limit DEH. The implementation of austerity pushes a country beginning at point A into insolvency, as illustrated in Figure 4. Point A is below the effective fiscal limit at the initial value of γ, but above the effective fiscal limit when γ increases.

Now we consider the case where the initial γ is small enough such that $\tau^H < \tau_t^{\max}$. In Figure 5, an increase in the tax adjustment parameter rotates the $\Delta\tau = 0$ curve clockwise around point G and moves point H in the northeast direction $\left(\frac{\partial \tau^H}{\partial \gamma} > 0,\ \frac{\partial b^H}{\partial \gamma} > 0\right)$, reducing

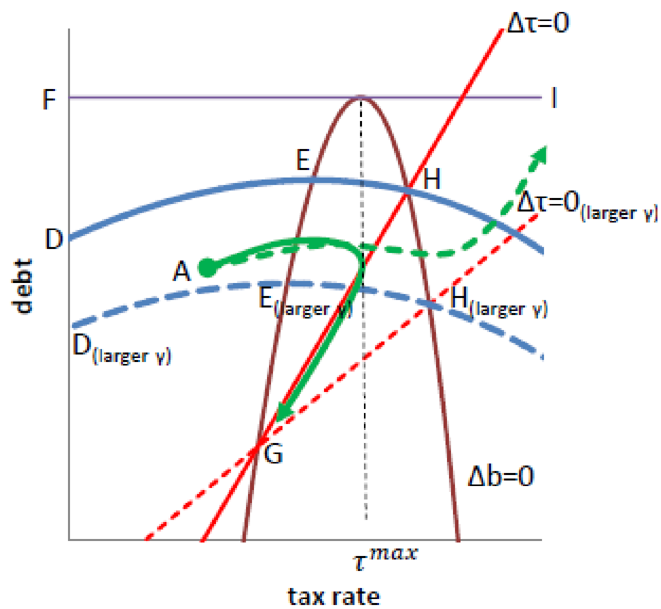

Figure 4: Lower effective fiscal limit

the area in which the dynamics become explosive in favor of an increase in the area in which debt converges to its long-run equilibrium. In this case, the increase in γ raises the effective fiscal limit DEH and reduces the likelihood a solvency crisis.

Our results imply that the efficacy of austerity depends on the initial value of the tax adjustment parameter. Austerity could be successful in averting a solvency crisis when the value of γ is small, whereas austerity could cause a solvency crisis when the value of γ is already large. Countries with large initial tax responsiveness to debt (γ) that choose to implement austerity are more prone to find themselves on the slippery side of the Laffer curve, where higher tax rates lower the tax revenue and the effective fiscal limit.[9]

2.8 Solvency crisis resolved with default

The maximum value of debt consistent with solvency is given by equation (11). When the economy moves above the effective fiscal limit, agents refuse to lend, creating a solvency

[9] Note that Bi's (2012) fiscal limit (horizontal line FI) remains the same regardless of the intensity of auterity, whereas Ghosh et al.'s (2013) debt limit increases as austerity intensifies.

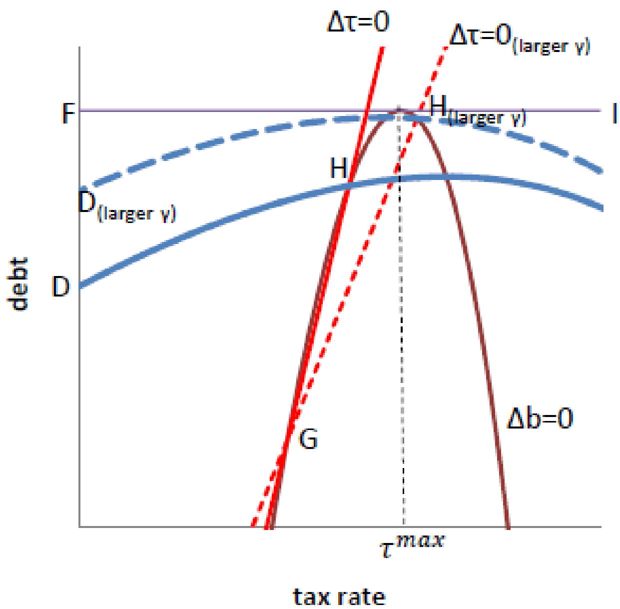

Figure 5: Higher effective fiscal limit

crisis. To restore lending, the government partially defaults. The magnitude of defaults returns debt back to the effective fiscal limit. We can write the fiscal space, Ω_t, between the effective fiscal limit, equation (11), and the current value of debt, equation (3), as

$$\Omega_t = \hat{b}_t - b_t = x_{t-1} + u_t + a_t - E_{t-1}a_t \tag{15}$$

where x_{t-1} is the difference between the effective fiscal limit and current debt in the absence of any shocks and defaults, and is given by

$$\begin{aligned}
x_{t-1} \quad = \quad & \hat{b}_{t-1} + \hat{\zeta}_{t-1}\left(\left(\rho^{\tau}-1\right)\left(\tau_{t-1}-\tau\right)+\gamma\left(b_{t-1}-b\right)\right)-\left(1+i\right)b_{t-1} \qquad (16) \\
& + \frac{\left(\tau\left(1-\rho^{\tau}\right)-\gamma b+\rho^{\tau}\tau_{t-1}+\gamma b_{t-1}\right)\left(\left(1-\rho^{A}\right)A+\rho^{A}A_{t-1}\right)\left(1-\tau\left(1-\rho^{\tau}\right)+\gamma b-\rho^{\tau}\tau_{t-1}-\gamma b_{t-1}\right.}{1+\phi-\left(1+\phi\lambda\right)\left(\tau\left(1-\rho^{\tau}\right)-\gamma b+\rho^{\tau}\tau_{t-1}+\gamma b_{t-1}\right)} \\
& + \frac{\left(\tau\left(1-\rho^{\tau}\right)-\gamma b+\rho^{\tau}\tau_{t-1}+\gamma b_{t-1}\right)\left[\phi\left(1-\lambda\right)\left(\left(1-\rho^{g}\right)g+\rho^{g}g_{t-1}\right)-\phi\lambda\left(\left(1-\rho^{z}\right)z+\rho^{z}z_{t-1}\right)\right]}{1+\phi-\left(1+\phi\lambda\right)\left(\tau\left(1-\rho^{\tau}\right)-\gamma b+\rho^{\tau}\tau_{t-1}+\gamma b_{t-1}\right)} \\
& - \left(\left(1-\rho^{g}\right)g+\rho^{g}g_{t-1}\right)-\left(\left(1-\rho^{z}\right)z+\rho^{z}z_{t-1}\right)
\end{aligned}$$

and u_t is the total impact of the fiscal and productivity shocks on the fiscal space

$$u_t = \frac{\left(\tau\left(1-\rho^\tau\right)-\gamma b+\rho^\tau\tau_{t-1}+\gamma b_{t-1}\right)\left[\varepsilon_t^A\left(1-\tau\left(1-\rho^\tau\right)+\gamma b-\rho^\tau\tau_{t-1}-\gamma b_{t-1}\right)+\phi\left(1-\lambda\right)\varepsilon_t^g-\phi\lambda\varepsilon_t^z\right]}{1+\phi-\left(1+\phi\lambda\right)\left(\tau\left(1-\rho^\tau\right)-\gamma b+\rho^\tau\tau_{t-1}+\gamma b_{t-1}\right)} -\varepsilon_t^g-\varepsilon_t^z.$$

A solvency crisis occurs if $\Omega_t < 0$. Negative shocks ($u_t < 0$), expectations of default ($E_{t-1}a_t > 0$), and changes in fiscal policy such as the implementation of strict austerity ($x_{t-1} < 0$), could push the economy over the effective fiscal limit, $\Omega_t < 0$. Default ($a_t > 0$) restores equilibrium.[10]

3 Model Applied: The case of Italy

In this section, we apply the model to Italy. First, we estimate the effective fiscal limit implied by our model, and we compare our results with Bi's (2012) fiscal limit and Ghosh et al.'s (2013) debt limit. Second, we quantify the probability of a solvency crisis when austerity is implemented.

3.1 At what point could Italy become insolvent?

Here, we quantify the Italian effective fiscal limit. The model is calibrated at annual frequency. We calibrate fiscal parameters to match the average of the Italian annual data between 1970 and 2015.[11] In the steady state, the tax rate (τ) is 0.40, the government spending is 18% of GDP, and the transfer payments are 20% of GDP, yielding a steady state debt level of 50% of GDP when the discount factor is set to deliver an annual real interest rate (i) of 4%. The fraction of foreign-held debt (λ) to total debt is 1/3, which is

[10] All the details are available in the appendix.

[11] All the variables are from the OECD database (OECD Economic Outlook No. 97). For τ_t we use the total revenue relative to GDP, for g_t we use the government final current consumption expenditure, and for z_t we use the sum of social security payments and subsidies.

the historical average over the sample period. Using least squares, the estimate for the tax responsiveness parameter (γ) is 0.34, and the tax rate persistence (ρ^{τ}) is 0.63. Using an HP filter, we detrend real GDP per worker, real government spending and real transfer payments to estimate the AR(1) processes for A_t, g_t and z_t. The estimates for the persistence are $\rho^{A} = 0.44$, $\rho^{g} = 0.69$ and $\rho^{z} = 0.31$, and the estimates for the standard deviation are $\sigma^{A} = 0.015A$, $\sigma^{g} = 0.025g$, and $\sigma^{z} = 0.024z$. The leisure preference parameter ϕ is set to 2.21 such that the household spends 25% of time working. The total amount of time and the productivity level at the steady state (A) are normalized to 1.

Table 1: Calibration to the Italian economy

Parameters	Value
Discount factor (β)	0.9615
Labour ($1-l$)	0.25
Leisure preference parameter (ϕ)	2.21
Technology (A)	1
Tax rate (τ)	0.40
Government spending/GDP (g/y)	0.18
Transfers/GDP (z/y)	0.20
Total debt/GDP (b/y)	0.50
Foreign held debt/total debt (λ)	0.33
Persistence of taxes (ρ^{τ})	0.63
Tax adjustment (γ)	0.34
Persistence of technology $\left(\rho^{A}\right)$	0.44
Standard deviation of technology $\left(\sigma^{A}\right)$	$0.015A$
Persistence of government spending (ρ^{g})	0.69
Standard deviation of government spending (σ^{g})	$0.025g$
Persistence of transfers (ρ^{z})	0.31
Standard deviation of transfers (σ^{z})	$0.024z$

Our model yields a hump-shaped effective fiscal limit ranging from 162% of GDP when the tax rate is close to zero (point D), to 194% of GDP when the tax rate is 0.52 (point E), to 188% of GDP when the tax rate is 0.72 (point H), as shown in Figure 6.[12] To determine the current fiscal state of the Italian government, we compute the fiscal space, equation (15),

[12]The debt levels are scaled by the steady-state output (y).

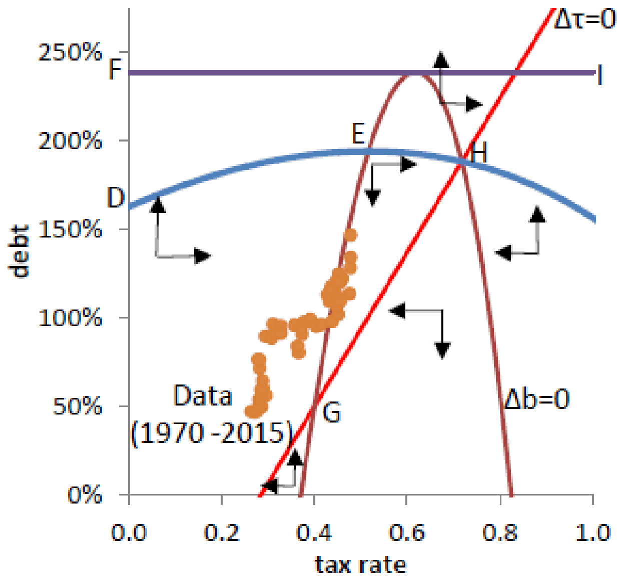

Figure 6: Italian effective fiscal limit

using the 2015 values of debt and tax rate. We find that in 2015 the Italian debt was below its effective fiscal limit, however the fiscal space has been declining over the last 35 years, as illustrated in Figure 7.

Next, we compute Bi's (2012) fiscal limit. Using equations (12) and (13), $\tau_t^{\max} = 0.62$ and Bi's procedure yields a $\hat{b}_t^{Bi}$ of 239% of GDP (horizontal line FI), as shown in Figure 6.[13] To compute Ghosh et al. (2013) debt limit, we first estimate a cubic model using least squares and annual data from 1970 to 2015. The estimates are presented in Table 2.[14] The coefficient on b_{t-1}^3 is negative, but enters significantly at the 99 percent level when lagged surplus, s_{t-1}, is excluded.[15] Based on the estimates from Regression (1), the marginal response of primary surplus to lagged debt $\left(\frac{\partial s_t}{\partial b_{t-1}}\right)$ begins to decline when debt exceeds 105%

[13]Bi's procedure yields a distribution. The estimate of 239% of GDP can viewed as the mean of the distribution.

[14]All the variables are from the OECD database (OECD Economic Outlook No. 97). For s_t we use the general government primary balances relative to GDP, for b_t we use the general government gross financial liabilities relative to GDP, for *output gap* we use the economy's output gap, and for *spending gap* we use the cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter.

[15]Lagged surplus captures persistence.

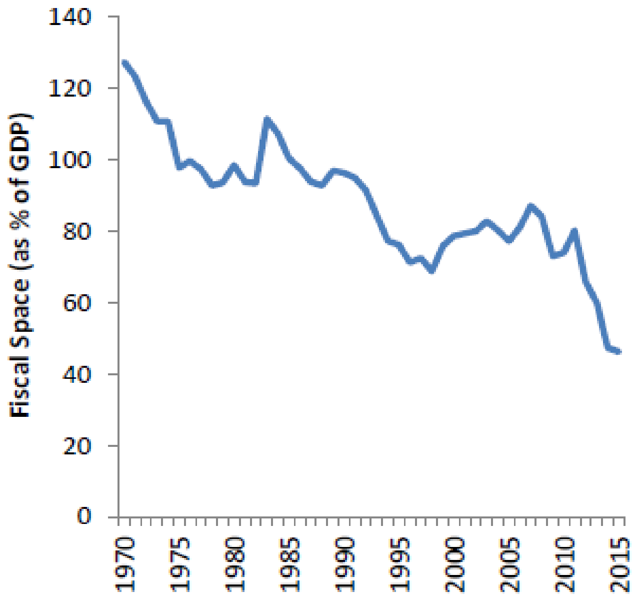

Figure 7: Fiscal Space

of GDP and becomes negative once debt exceeds 140% of GDP.[16] To compute $\hat{b}^{Ghosh}$ we use the fiscal policy parameters from Regression 1, and an interest rate. Following Ghosh et al. (2013), we use average growth adjusted interest rate over the last ten years which is 3%. Similar to their results we are unable to obtain an estimate for the Italian debt limit using their approach because the interest payments are always above the estimated cubic function, as shown in Figure 8. The Ghosh et al. (2013) procedure suggests that the Italian debt has already breached its debt limit and is on an explosive path, without actually yielding an estimate of what the Italian debt limit is.

[16]The magnitudes are similar to those estimated by Ghosh et al. (2013).

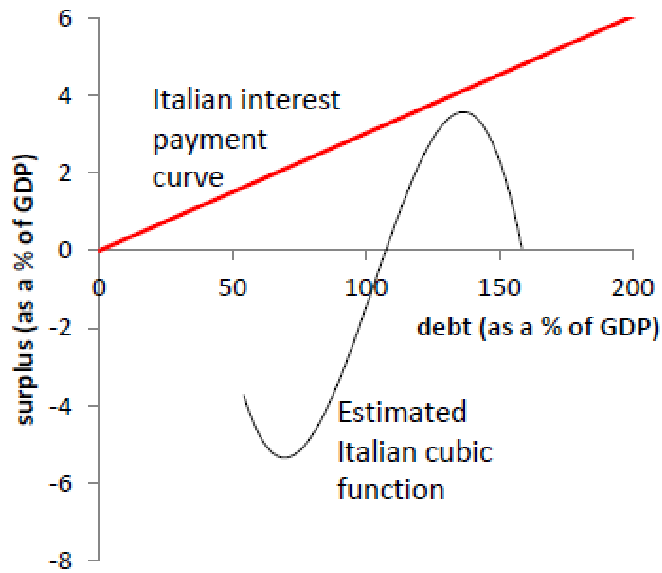

Figure 8: Estimated cubic function

Table 2: Estimates of the cubic fiscal rule

	(1)	(2)
c	36.8213***	10.8086
	(9.7867)	(10.2374)
b_{t-1}	-1.4857***	-0.5327
	(0.3025)	(0.3386)
b_{t-1}^2	0.0163***	0.0063*
	(0.0030)	(0.0030)
b_{t-1}^3	-0.00005***	-0.00002*
	(0.000009)	(0.00001)
output gap	0.2175**	0.1591*
	(0.0030)	(0.0838)
spending gap	-0.0884	-0.0758
	(0.0626)	(0.0525)
s_{t-1}		0.4969***
		(0.1171)
$\bar{R}^2$	0.8356	0.8847

Note: The *, ** and *** indicate statistical significance at the 90, 95 and 99 percent level, respectively.

To summarize, the Ghosh et al. approach cannot provide an estimate for the Italian debt limit. Bi' approach suggests that the point that Italy could become insolvent is around 239% of GDP regardless of the tax rate. Our approach yields a time varying effective fiscal limit, which depends on the tax rate, and we find that at the current level of debt and taxes, Italy is not on an explosive path.

3.2 Simulations

In this section, we quantify the probability of a solvency crisis. To estimate the probability of a solvency crisis, we use 1000 replications of a 10 year simulation. For the simulations, we use the parameter values from Table 1, and the simulation algorithm in Table 3. We assume that all shocks have a normal distribution with mean zero, and we set a lower and upper bound on the fiscal and productivity shocks to correspond to two standard deviations.[17]

While Italy is below its effective fiscal limit, it could still experience a solvency crisis because of negative shocks. To consider the impact of stochastic shocks on the current Italian fiscal state, we simulate the model using the 2015 values of the tax rate and debt.[18] We find that under the baseline parameters values, the probability of a solvency crisis in Italy is zero. The 2015 value of Italian debt is well below its effective fiscal limit that there should be no concerns about a solvency crisis.

Next we estimate the impact of austerity on solvency crisis. We consider how the probability of a solvency crisis changes as γ increases beyond its baseline value of 0.340. Figure 9 plots the probability of a crisis as a function of γ. We find that if γ increases by two standard deviations to 0.408, the probability of a crisis remains zero. The crisis probability becomes

[17]We set bounds on the distributions of the shocks to avoid skewing the results with draws close to $\pm\infty$.

[18]Source: $\tau_{t-1} = 0.48$ and $b_{t-1} = 147\%$ of GDP. OECD Economic Outlook No. 97

positive once γ exceeds 0.573, and unity once γ exceeds 0.590. Our results suggest that the adoption of aggressive austerity, which doubles the rate at which the Italian government retires debt, could push Italy into insolvency. The opposite also holds. A country with a large γ that eases up on austerity could regain access to the markets and re-attain a solvent position.

Next we consider whether the probability of a crisis changes as γ declines from its baseline value of 0.340. We repeat the simulations and find that the crisis probability becomes positive once γ falls below 0.109. Our results suggest that whether or not austerity could alleviate or cause a solvency crisis depends on the country's starting value of γ. The probability of crisis does not monotonically declines as γ increases. For very low values of γ, an increase in the responsiveness lowers the probability of a solvency crisis; however for high values of γ, an increase in the tax adjustment parameter raises the probability of a solvency crisis. This result could shed some light on the ongoing debate in Europe about austerity. Pressure on high-debt countries by Troika (EMU and IMF) to adopt strict austerity measures could indicate the perception that high-debt countries like Italy have a small baseline value γ, whereas the resistance by the high-debt countries to adopt strict austerity could indicate their judgement that they have a large enough γ such that further increases in γ will result to insolvency.

To illustrate how sensitive the probability of a crisis to austerity, we repeat the simulations by changing other values one at a time in the risky direction. Our sensitivity scenarios include: (1) lower initial tax rate (τ_{t-1}), (2) higher persistence in taxes (ρ^{τ}), and (3) lower fraction of foreign held debt (λ). The experiments illustrate three interesting implications of our model. First, the hump-shaped effective fiscal limit implies that if Italy's 2015 value of

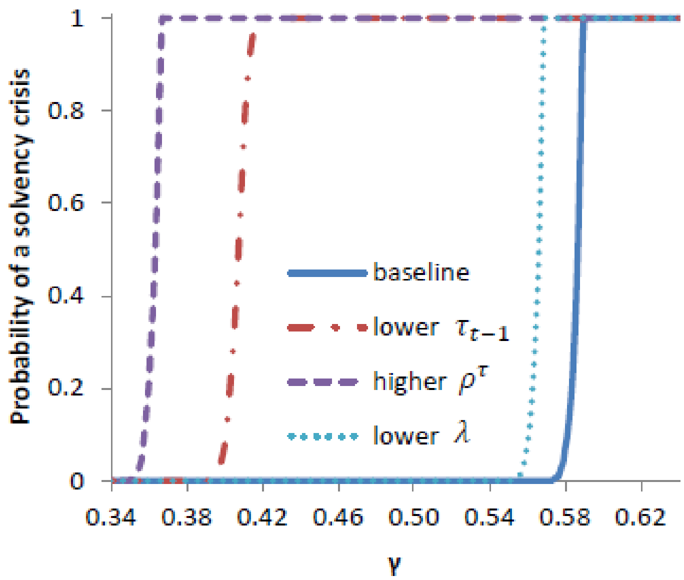

Figure 9: Probabilities of a solvency crisis as a function of the tax adjustment parameter.

initial tax rate (τ_{t-1}) had been smaller than 0.48, then the adoption of modest austerity would have substantially increased the probability of a crisis. When τ_{t-1} is set to the minimum tax rate in our sample of 0.284, the crisis probability becomes positive once γ exceeds 0.391, and unity once γ exceeds 0.420. Second, our results imply that higher persistence in taxes, most likely stemming from the rigidities or difficulties in changing taxes frequently, substantially increases crisis probabilities. When ρ^{τ} increases by two standard deviations from 0.64 to 0.80, the crisis probability becomes positive once γ edges up from 0.34 to 0.35, and unity once γ exceeds 0.367. Third, the results imply that a decline in the fraction of government debt held by foreign agents increases the probability of solvency crisis. When λ declines from 0.33 to the minimum value in our sample of 0.175, the crisis probability becomes positive once γ exceeds 0.553. Although for most countries the true λ is less than one, a common simplifying assumption in the sovereign strategic default literature is that government's debt is 100% foreign held ($\lambda = 1$). Our results suggest that models that assume $\lambda = 1$, when the true $\lambda < 1$, underestimate the likelihood of future insolvency.

In summary, while the crisis probabilities are higher under Experiments 1-3 as shown in Figure 8, the implications are identical to the ones under the baseline parameters. Countries with very small γ could increase their effective fiscal limit and lower the probability of a solvency crisis as γ increases, while countries such as Italy could lower their effective fiscal limit and raise the probability of a solvency crisis as they intensify austerity.

4 Conclusion

Many countries are implementing strict austerity measures, whereby governments aggressively raise taxes. In this paper, we show that austerity affects the probability of a solvency crisis. First, we endogenously derive the maximum level of debt consistent with solvency, which we call the effective fiscal limit on debt. Second, we show that austerity affects the position of the effective fiscal limit. We find a nonlinear relationship between austerity and the effective fiscal limit. For very small values of the tax adjustment parameter, austerity could raise the effective fiscal limit and prevent a solvency crisis. For any other values, austerity could lower the effective fiscal limit and induce a solvency crisis. Third, we apply the model to Italy, a country that is under ongoing pressure from the IMF and EMU to rein in its rising debt level. We estimate the Italian fiscal limit and quantify the impact of intensifying austerity on the probability of solvency crisis. Should Italy implement aggressive austerity, our model warns of a potential solvency crisis.

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5 Appendix A: Solvency crisis resolved with default

We define a shadow value of capital loss via default, $\tilde{a}_t$, which represents the reduction in the value of debt needed for the economy to reach equation (11). Setting Ω_t to zero in equation (15) yields

$$\tilde{a}_t = E_{t-1}a_t - x_{t-1} - u_t. \tag{17}$$

Substituting into equation (15) yields an expression for Ω_t as

$$\Omega_t = a_t - \tilde{a}_t.$$

When the shadow value is positive of capital loss via default is positive ($\tilde{a}_t > 0$), default equal to the shadow value sets $\Omega_t = 0$ and restores solvency. When the shadow value is negative, there is no default.

To solve for the magnitude of default, a_t, we must first solve for expectations of default, $E_{t-1}a_t$.Define u_t^* as a critical value for the shock u_t such that $a_t > 0$ for $u_t < u_t^*$, and $a_t = 0$ for $u_t \geq u_t^*$. Letting $f(u_t)$ be a bounded, symmetric, mean-zero distribution for u_t, with bounds given by $\pm\,\bar{u}$, the probability of a solvency crisis can be expressed as

$$F(u_t^*) = \int_{-\bar{u}}^{u_t^*} u_t f(u_t)$$

and the expectation for (17) can be written as

$$E_{t-1}a_t = \int_{-\bar{u}}^{u_t^*} a_t f(u_t) = \int_{-\bar{u}}^{u_t^*} (E_{t-1}a_t - x_{t-1} - u_t)\, f(u_t)\,.$$

Collecting terms on the expectation of default yields

$$[1 - F(u_t^*)]\, E_{t-1}a_t = -x_{t-1}F(u_t^*) - \int_{-\bar{u}}^{u_t^*} u_t f(u_t)\,. \tag{18}$$

Substituting into equation (17), yields an implicit expression for a_t

$$[1 - F(u_t^*)]\, a_t = -\left[x_{t-1} + u_t\,(1 - F(u_t^*)) + \int_{-\bar{u}}^{u_t^*} u_t f(u_t)\right]. \tag{19}$$

There is a solution for u_t^* iff $x_{t-1} \geq 0$. For large positive values of x_{t-1}, the critical value of the shock (u_t^*) equals its lower bound ($-\bar{u}$). As x_{t-1} falls, u_t^* rises, reaching its upper bound at $\bar{u}$ once $x_{t-1} = 0$. For negative values of x_{t-1}, even the upper support ($\bar{u}$) does not satisfy the equation (19) because when $u_t^* = \bar{u}$, the left-hand side of the equation (19) is zero and the right-hand side is positive. Therefore, existence of an equilibrium value for expected default requires that $x_{t-1} \geq 0$.

6 Appendix B: Simulation algorithm

Table 3: Simulation Algorithm

Probability of a solvency crisis over the next ten years
1. Compute the state variable determining the fiscal space, x_{t-1}, from equation (16) using initial values of debt, b_{t-1}, tax rate, τ_{t-1}, government spending, g_{t-1}, transfers, z_{t-1}, and technology, A_{t-1}.
2. Compute the expectations for default, $E_{t-1}a_t$, from equation (18).
3. Draw a productivity shock, ε_t^A, a government spending shock, ε_t^g, and a transfer shock, ε_t^z.
4. Calculate the value for capital loss due to default, a_t, from equation (19).
5. If $a_t > 0$, then there is a solvency crisis and the simulation ends.
6. If $a_t = 0$, then next period's debt, taxes, government spending, transfers and productivity are updated which are then used to update x_t.
7. Repeat steps 2-6 for ten years.
8. Repeat the ten-year simulation 1000 times. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.