LCERPA
Laurier Centre for Economic Research & Policy Analysis

LCERPA Working Paper No. 2019-2

Original: October 2019
Revised: August 2021

Identifying Countries at Risk of Fiscal Crisis: High-Debt Developed Countries

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May 27, 2021

Abstract

Crises in European countries in 2010 and beyond demonstrated that fiscal crises and sovereign default are not confined to emerging and developing countries. Advanced economies can sustain much larger debt-to-GDP ratios than emerging economies, but how much larger? Experience is heterogeneous both across countries and across time. What determines this heterogeneity? We show that a low growth-adjusted interest rate, a large maximum value for the primary surplus, and a strong surplus-responsiveness to debt can support higher debt-to-GDP ratios without fiscal crisis. We use our estimates to assess fiscal crisis risk for nine-high-debt developed countries following the financial crisis in 2008. Our results imply that Ireland and Portugal lost access to financial markets due to the rise in growth-adjusted interest rate, whereas Greece would have lost access regardless of the interest rate. Additionally, our results warn of potential future crises for Greece, Italy and Japan even if these countries remain in a low interest rate environment.

JEL Classifications: E62, F34

Keywords: Fiscal Limits, Fiscal Space, Fiscal Rules, Fiscal Solvency, Sovereign Default

*We thank the editor Viktoria Hnatkovska and three anonymous referees for helpful comments. We also thank seminar participants at the 2016 Meetings of the Canadian Economic Association, the 2015 Meetings of the Royal Economic Society, the 2015 Meetings of the Western Economic Association International, and the University of Cyprus. Research funding provided by the Money Macro and Finance Research Group, the Wilfrid Laurier University, and the Social Sciences and Humanities Research Council of Canada.

1 Introduction

How large can government debt get before a country looses access to financial markets? What determines the maximum value of debt beyond which a country loses access? Why do countries exhibit heterogeneity in debt tolerance? Most experience with fiscal crises has been with emerging and developing countries, in which crises have occurred with much lower debt ratios.[1] Advanced countries can accumulate more debt without a crisis, but how much more? Greece lost access to credit markets when the debt-to-GDP ratio exceeded 130%. Portugal and Ireland lost access with debt-to-GDP ratios of 100% and 90%, respectively. These same countries regained access later with much higher debt ratios. In contrast, Belgium experienced a similar debt ratio to Greece and never lost access. We use data from 1980-2019 to analyze risk in nine high-debt, advanced countries, three of which lost access to financial markets in early 2010's and re-gained access years later. The high-debt countries include Belgium, Canada, France, Greece, Ireland, Italy, Japan, Portugal and Spain.

We consider fiscal crises due to insolvency.[2] We assume that a developed country is committed to honor its debt obligations when it can, and that it faces a limit on the value of the primary surplus as a fraction of GDP, referred to as the surplus limit. Defining debt and primary surpluses as values relative to GDP, a government's intertemporal budget constraint requires that the country is expected to raise future primary surpluses, whose present value is sufficient to repay outstanding debt. The largest debt a country can repay, equivalently the debt limit, is the present value of expected future surpluses. If the surplus limit keeps the present value of expected future primary surpluses from being large enough, then default is necessary to reduce the value of debt. The probability of a crisis one period ahead is the probability that a shock next period raises debt and/or reduces the expected present value of future surpluses sufficiently to require default to satisfy the intertemporal budget constraint.

To implement our assessment of fiscal risk empirically, we need to estimate the debt limit, which in turn requires a forecast of the expected path of future primary surpluses and a measure of the interest rate to discount these future values. To forecast expected

[1] Arellano et al. (2020) choose Argentina, Brazil, Ecuador, Mexico, Peru, South Africa, Russia, and Turkey as emerging and developing countries with limited fiscal space. Their average debt-to-GDP between 2001 and 2019 is 44%. The average debt-to-GDP of the nine high-debt countries in our sample is 106% over the same period.

[2] There are other reasons for default, including strategic default, self-fulfilling default due to expectations, and default due to illiquidity. Each would require a different model to answer our lead questions. In this paper, we build a solvency model to answer our lead questions.

future values of the primary surplus, we estimate a fiscal feedback rule (Bohn 1998, 2007; Leeper 2010), allowing the primary surplus to respond to its own lagged value, to the value of lagged debt, and to other business-cycle indicators. We use the estimated coefficients, together with initial values for the debt and surplus and a value for the discount factor, to compute the expected present value of future surpluses.

The appropriate interest rate for discounting the path of expected future surpluses is the growth-adjusted interest rate, approximately the difference between the nominal interest rate and the growth rate of nominal GDP. Growth-adjusted interest rates for all countries in our sample have experienced sharp swings since 1980. In the early-1980's, growth-adjusted interest rates rose, with a sharp increase in nominal interest rates. In the late 1990's, reductions in nominal rates led to a decline in growth-adjusted interest rates, and they have remained low for most countries. The IMF (2018) has argued that the value for the growth-adjusted interest rate is a key determinant of fiscal crises.

We explicitly consider the effect of different values of the growth-adjusted interest rate on the debt limit. To keep the model tractable, we assume that there are two interest rate regimes, one with a high rate and one with a low rate. This assumption is supported using the Bai and Perron (1998, 2003) break tests for growth-adjusted interest rates in each country. There are two ways in which the growth-adjusted interest rate affects the debt limit. First, a lower value implies that future maximum values for primary surpluses are discounted at a lower rate, yielding a higher debt limit. Second, we find that the value for the growth-adjusted interest rate also affects the estimated coefficients in the fiscal feedback rule used to forecast future primary surpluses.

We investigate the likelihood of a fiscal crisis one period ahead, conditional on a particular interest rate regime. The intertemporal budget constraint, together with the fiscal feedback rule, requires that the value for current debt equal the expected present value of primary surpluses. The country is solvent without default if the value of the primary surplus, everywhere along the expected path determined by the fiscal feedback rule, is below the surplus limit. The expected path for primary surpluses, which peaks at the surplus limit, contains the largest possible future primary surpluses. Therefore, the largest value of debt allowing solvency, the debt limit, is the present value of the expected surplus path whose peak is at the surplus limit. Since the path of expected future surpluses depends on the current surplus, the debt limit also depends on the current surplus. Insolvency occurs if outstanding debt exceeds the debt limit. Default, defined as a reduction in the level of debt that will be repaid, restores solvency.

A country is at risk of a fiscal crisis if adverse shocks next period could either raise debt and/or reduce the debt limit such that debt exceeds its debt limit.[3] We define fiscal space as the largest increase in debt possible without a fiscal crisis. We use our fiscal space measures to assess risk of a fiscal crisis in each year between 2008 and 2019. For each year, we place countries into three risk categories, "safe", "risky" and "highest risk". We label a country as "safe" if fiscal space is so large that the probability of receiving shocks large enough to require default is virtually zero. Alternatively, the country is at "highest risk" if it has exhausted its fiscal space. Intermediate values of fiscal space place countries in the "risky" category, with risk increasing at an increasing rate as fiscal space shrinks (Daniel and Shiamptanis 2012).

Our risk assessment is conditional on the interest rate regime. Using Hamilton's (1989) regime switching model for the growth-adjusted interest rate, we find small probabilities of switching between high and low interest rate regimes. This implies that a country, which appears "safe" in a low-interest-rate regime, but at "highest risk" in a high-interest-rate regime, could experience a fiscal crisis due to the probability of switching to the high-interest rate regime. This highlights the importance of the interest rate in evaluating the likelihood of a fiscal crisis. Models which assume a single interest rate regime miss these issues.

The contribution of the paper is four-fold. First, we present a new approach to deriving the debt limit. The standard approach sets the debt limit as the present value of a fixed surplus limit, requiring that a country which reaches the debt limit raise the primary surplus to the surplus limit and maintain it forever.[4] However, this behavior is not in the data. Austerity forever is not politically viable. Our approach yields a lower debt limit than the standard procedure. Additionally, our approach yields a debt limit which is a function of the current surplus since the path of expected future surpluses depends on the current surplus.

Second, we allow the interest rate regime to affect the debt limit in two ways, first, by affecting estimates of the expected present value of maximum future surpluses, and second by affecting the value of the interest rate used to discount future expected surpluses. Different debt limits, conditional on different interest rate regimes, allow us to explain why some European countries experienced crises in the early 2010's as growth-adjusted interest rates spiked, but regained access in the mid-2010's at higher debt ratios, as growth-adjusted interest rates declined.

Third, we provide empirical estimates of fiscal space for each of our nine high-debt

[3]A reduction in the surplus reduces the expected present value of future surpluses reducing the debt limit.

[4]Tanner (2013), Collard et al. (2015), Bi (2012), and Davig et al. (2011) are examples.

developed countries for each year after 2008, conditional on each interest rate regime, and use these estimates to assess the risk of a fiscal crisis faced by each country in each year after 2008. Our model places Greece, Ireland, and Portugal in the "highest risk" category, predicting the crises which occurred. Countries in the "safe" category, Belgium and Canada, did not experience crises. Other countries have switched categories and face potential crises after our end date of 2019.

Finally, we offer answers to our initial questions about why countries exhibit heterogeneity in debt tolerance. The level of debt which triggers a crisis, the debt limit, is heterogenous across time and across countries. The time variation in debt limits is due to changes in the interest rate regime over time with a low interest regime yielding a higher debt limit. The country variation in debt limits is due to differences in surplus limits, coefficients on the estimated fiscal feedback rules, and initial values for the primary surpluses. Therefore, our debt limit depends on the surplus limit, current value for the primary surplus, the coefficients of the fiscal feedback rule, and the interest rate regime.

We should also state what our paper does not do. The literature has many reasons a country could default besides solvency. Crises occur when there is too little liquidity even though the country is solvent. We do not model liquidity crises, equivalently rollover crises, for which debt maturity is important. We are also not considering strategic default, in which a sovereign has no commitment to repay and does so only when the benefits exceed the costs. Therefore, our model does not include all possible causes of default (IMF 2018). Our focus is narrower – default due to insolvency.

Our work builds on that of others who have used debt limits to assess insolvency risk. The debt limit has been constructed using a limit on the primary surplus, determined either with a Laffer curve or a historical maximum, and then applying the government's intertemporal budget constraint, with the path for expected future primary surpluses fixed at the surplus limit (Tanner 2013; Collard et al. 2015; Bi 2012; Davig et al. 2011). In their framework, the debt limit is the present value of the surplus limit. Collard et al. (2015) modify this analysis for the interest premium created by the probability of default. They calculate maximum sustainable debt as the value of debt, with the surplus at its limit, beyond which debt becomes explosive due to its default premium. Our measure of the debt limit, as the expected present value of the path of future surpluses, whose peak is at the surplus limit, implies a lower value for the debt limit and does not require agents to expect a policy of austerity forever with the surplus fixed at its limit.

Ghosh et al. (2013) offer a measure of a debt limit based on the assumption that future surpluses are determined by a nonlinear function of powers of debt, where the function is homogeneous across countries. In their panel model, the cubic power of debt has a negative coefficient, reflecting "fiscal fatigue," enabling them to use growth-adjusted interest rates to identify values of debt beyond which debt explodes. Since no creditor would lend to a borrower whose debt is explosive, their debt limit is the value of debt at which the system becomes unstable. Their approach requires that countries actually reduce (not just fail to increase) their primary surplus as debt increases beyond very large values, a data point experienced by very few countries in their sample. In their model, the only heterogeneity across countries is the country-specific interest rate. We allow the fiscal policy response to debt to be heterogenous across countries, behavior we find important in explaining different tolerance to debt.

This paper also extends Daniel and Shiamptanis (2013), which assesses the probability of a fiscal crisis for Greece where the crisis response can either be default or policy switching from active monetary, passive fiscal (AMPF) to passive monetary, active fiscal policy (PMAF). In the current paper, we restrict the policy response to default, allowing us to extend the model in other ways, specifically by endogenizing the debt limit and highlighting the roles of the growth-adjusted interest rate and the initial value of the surplus on the debt limit. We use our extended default model to study fiscal risk in nine high-debt developed countries.

Celasun et al. (2007) provide an estimate of long-run fiscal risk in contrast to our estimate of short-run risk. They estimate a fiscal feedback rule and interact estimated coefficients with the value of expected future shocks based on an estimated VAR, to predict probability bounds on future values of debt. Debt is viewed as non-sustainable if these probability bounds indicate a high probability of rising debt over time. In our model, expected future debt does not have a monotonic path, and fiscal risk can be low even if debt is expected to rise in the near term.

This paper is organized as follows. The next section derives the debt limit and fiscal space. Section 3 is empirical, with estimates of fiscal feedback rules. Section 4 uses our estimates to measure fiscal space and assess the risk of a fiscal crisis following the global financial crisis for our nine high-debt developed countries. Section 5 provides conclusions.

2 Model

We develop a partial equilibrium model of fiscal policy, in which we treat the non-fiscal part of the economy as exogenous. We assume that there are two interest rate regimes, one with a high rate and one with a low rate. The two rates are exogenous and do not vary within each regime. We present the model for a particular country, conditional on a particular interest rate regime, indexed by a subscript h. Our solution is conditional on remaining in interest regime h. The model of fiscal policy has four key components, the dynamic behavior of the surplus, the surplus limit, the government's budget constraint, and the arbitrage equation determining the equilibrium interest rate, conditional on the interest rate regime. We consider an exogenous possibility of switching between the two interest rate regimes at the end.

2.1 Dynamic Behavior of the Surplus

Our fiscal feedback rule for the surplus (s_t) extends those originally proposed by Leeper (1991) and Bohn (1998) by adding a lagged surplus term (s_{t-1}) and by allowing the constant (c_h) and the responsiveness of the surplus (γ_h) to lagged debt (d_{t-1}) to take different values in each regime h. The fiscal feedback rule is given by

$$s_t = c_h + \gamma_h d_{t-1} + \beta s_{t-1} + \epsilon_t \qquad \gamma_h > 0,\ h \in \{1, 2\}\,, \tag{1}$$

where all variables are expressed relative to GDP. The parameter β measures the persistence in the primary surplus stemming from inertia in the legislative process, and we assume that it is constant across regimes.[5] Think about persistence as due to the fact that legislatures do not reevaluate all of their tax and spending plans every year, but instead, make adjustments at the margin, due to the evolution of debt and shocks. The parameter γ_h captures the primary surplus responsiveness to debt. A positive and large enough γ_h assures that the government's intertemporal budget constraint is expected to be satisfied in regime h.

The ϵ_t is an independent and identically distributed (iid) shock to the primary surplus with a bounded, zero-mean, symmetric distribution. It includes the output gap shocks ($\tilde{y}_t$) as well as transitory government spending ($\tilde{g}_t$), as in Bohn (1998, 2007), Mendoza and Ostry (2008) and Ghosh et al. (2013). Therefore,

$$\epsilon_t = \delta_1 \tilde{y}_t + \delta_2 \tilde{g}_t + \tilde{\epsilon}_t,$$

where $\tilde{\epsilon}_t$ denotes other shocks.

There is large literature on the Fiscal Theory of the Price Level (FTPL) which seeks

[5] We test this hypothesis for each country and cannot reject the null hypothesis of equality across the two regimes at the five and ten percent significance level for all countries, except Canada. For Canada, we reject at the five percent level, but cannot reject at the ten percent level.

to test whether fiscal policy is active or passive by estimating whether or not γ_h differs significantly from zero. Cochrane (1998) demonstrates that equilibrium under active monetary policy, passive fiscal policy (AMPF) is observationally equivalent to that under passive monetary policy, active fiscal policy (PMAF). Therefore, we do not have a way to empirically separate the two systems. Our maintained assumption is that $\gamma_h > 0$, equivalent to AMPF policy. Our empirical work is focused on determining its magnitude in each country under the assumption that its sign is positive. In the opposite case, PMAF, default is never necessary to restore solvency because the price level is determined to satisfy the government's expected intertemporal budget constraint at any value for debt. Therefore, we assume that policy is always AMPF restricting our analysis to systems in which default risk exists.

2.2 Surplus Limit

We assume that every sovereign faces a limit on its ability to raise government surpluses with tax increases and spending reductions, and therefore a limit on its ability to repay and service debt. Davig et al. (2011), and Bi (2012) motivate the surplus limit by the top of the Laffer curve for distortionary taxes. However, the concept can be more general. A limit on the surplus can be due to the inability to reduce government spending, perhaps due to the dependence of economic activity on the provision of public goods, and to the inability to raise tax rates for other reasons, including tax evasion (Daniel 2014). Bi et al. (2013) argue that the surplus limit could also be political, whereby the democratic process cannot raise the surplus sufficiently to service the debt. We denote the absolute maximum value of the primary surplus relative to GDP as $\bar{s}$.

We assume that there is an $\bar{s}$, and use historical data on primary surpluses and debt to estimate its magnitude. Since it is possible that a country's surplus has never reached its limit, our risk estimates acknowledge that the limit could be an underestimate.

2.3 Government's Budget Constraint

All debt is one-period debt and evolves according to the government's flow budget constraint, given by

$$d_t = \delta_t \left(\frac{1 + i_{t-1}}{1 + \rho_t} \right) d_{t-1} - s_t, \tag{2}$$

where i_{t-1} is the domestic interest rate for debt issued in period $t-1$, and ρ_t is the growth rate of domestic output in period t.[6] The δ_t denotes the fraction of debt that is repaid at time t, such that $\delta_t = 1$ implies no default, and $\delta_t < 1$ implies default.

2.4 Arbitrage Equation for Interest Rate

We present the model for one of the two interest rate regimes, indexed by h. The risk-free world interest rate (i_h) is exogenous and does not vary within regime h. We index values that do not vary in regime h with the h subscript, but suppress it on other variables. We assume that the government has access to international lenders who are willing to lend any amount to the domestic government as long as they expect to receive the risk-free world interest rate (i_h) in regime h. Given these assumptions, interest rate parity determines the domestic interest rate (i_{t-1}) according to

$$1 + i_h = (1 + i_{t-1})\, E_{t-1}\delta_t, \tag{3}$$

where $E_{t-1}\delta_t = 1$ implies that there no expectations of default and $E_{t-1}\delta_t < 1$ reflects expectations of default. When lenders expect default ($E_{t-1}\delta_t < 1$), the domestic interest rate (i_{t-1}) rises to offer risk-neutral lenders an expected rate of return equal to that in the market. The world risk-free rate is exogenous, but the country interest rate contains an endogenous risk premium when $E_{t-1}\delta_t < 1$.

2.4.1 Dynamic Behavior of Debt

In this section, we construct the growth-adjusted interest rate and substitute it into equation (2) to obtain the dynamic behavior of debt. To construct the growth-adjusted interest rate, assume that the inverse of the gross domestic growth rate $\left(\frac{1}{1+\rho_t}\right)$ is distributed iid about a regime-specific mean of $\frac{1}{1+\rho_h}$ such that

$$\frac{1}{1+\rho_t} = \frac{1}{1+\rho_h}\zeta_t, \tag{4}$$

where ζ_t captures stochastic growth shocks and $E_{t-1}\zeta_t = 1$. Using this assumption, define interest rates, adjusted by the mean domestic growth rate, as

$$(1 + r_h) = \frac{1+i_h}{1+\rho_h}, \qquad (1 + r_{t-1}) = \frac{1+i_{t-1}}{1+\rho_h}, \tag{5}$$

[6] We can view i_t and ρ_t either as nominal or real with no effect on the derivation.

where r_h (r_{t-1}) is the world interest rate in regime h (country interest rate at time $t-1$), adjusted by the mean growth rate for the country in regime h. Substituting interest rates from equations (5) into interest rate parity, equation (3), and dividing both sides by $1+\rho_h$ yields

$$1 + r_h = (1 + r_{t-1}) \, E_{t-1}\delta_t. \tag{6}$$

Define α_t as

$$\alpha_t = (1 - \delta_t) \, (1 + r_{t-1}) \, d_{t-1}, \tag{7}$$

where α_t has the interpretation as the capital loss due to default. Using equation (7) and substituting from equation (6), unexpected capital loss due to default can be expressed as

$$\alpha_t - E_{t-1}\alpha_t = \left[-\delta_t \, (1 + r_{t-1}) \, d_{t-1} + (1 + r_h) \, d_{t-1}\right]. \tag{8}$$

Using equations (4) and (5), the equation for the evolution of debt (2) becomes

$$d_t = \zeta_t \delta_t \, (1 + r_{t-1}) \, d_{t-1} - s_t. \tag{9}$$

The gross payments on debt $[\zeta_t \delta_t \, (1 + r_{t-1})]$ contains both growth shocks and potential loses from default. Substituting $\delta_t \, (1 + r_{t-1}) \, d_{t-1}$ from equation (8) into equation (9) yields

$$d_t = (1 + r_h) \, d_{t-1} - s_t - \eta_t + \psi_t, \tag{10}$$

where

$$\psi_t = [\zeta_t - 1] \, (1 + r_h) \, d_{t-1}, \tag{11}$$

$$\eta_t = [\alpha_t - E_{t-1}\alpha_t] \, \zeta_t. \tag{12}$$

We have decomposed the gross payments on debt into a term independent of time t shocks, $(1 + r_h) \, d_{t-1}$, and stochastic terms, η_t and ψ_t.

Equation (10) is an equation for the evolution of debt, which is linear in lagged debt, the primary surplus, and mean-zero stochastic terms. Equation (11) captures the impact of the growth shock on debt. An adverse growth shock $(\zeta_t > 1)$ raises the domestic growth-adjusted interest rate, increasing ψ_t, thereby raising debt. Equation (12) captures the impact of a default premium on debt due to expected capital loss, as well as unexpected capital loss on debt. The expectation of capital loss due to default $(E_{t-1}\alpha_t > 0)$ raises the domestic interest rate $(\eta_t < 0)$, thereby raising debt. Actual capital loss due to default $(\alpha_t > 0)$ reduces debt $(\eta_t > 0)$. When the expectation of default equals actual default, there is no debt reduction $(\eta_t = 0)$ because the interest rate fully adjusts to offset the future default.

Solving equation (10) J periods forward for debt, and imposing $\lim_{J\to\infty} \frac{d_{t+J}}{(1+r_h)^J} = 0$,[7] yields an expression for the government's intertemporal budget constraint, conditional on remaining in a particular interest rate regime, as

$$d_t = \sum_{k=1}^{\infty} \left[s_{t+k} + \eta_{t+k} - \psi_{t+k}\right] \left(\frac{1}{1+r_h}\right)^k . \tag{13}$$

Satisfaction of the government's intertemporal budget constraint, equation (13), does not require that the government never default, or equivalently that $\alpha_{t+k} = 0 \; \forall k$. Default can occur, but it provides revenue only if it is larger than its expected value, yielding $\eta_{t+k} > 0$. Therefore, systematic and expected default cannot provide revenue.

Taking the expectation of equation (13) yields the expected intertemporal budget constraint as

$$d_t = \sum_{k=1}^{\infty} \left(\frac{1}{1+r_h}\right)^k E_t\left(s_{t+k}|s_t, d_t\right), \tag{14}$$

validating that default cannot generate expected (and systematic) revenue. Satisfaction of the expected intertemporal budget constraint requires that the government be expected to generate future surpluses whose present value equals the value of debt, subject to the surplus limit.

If the government cannot raise large enough future surpluses to satisfy the expected intertemporal budget constraint (14), then the government is insolvent. We assume that default reduces the value of debt to satisfy equation (14). And since only unexpected default can eliminate an expected shortfall in equation (14), then if default is necessary in equilibrium, it must be unexpected. The possibility of default affects initial values and dynamics through shocks and expectations.

2.5 Equilibrium in Government Bond Market

Definition: Given the regime h values for the world interest rate (i_h) and the mean country growth rate (ρ_h), values for the lagged primary surplus (s_{t-1}) and debt (d_{t-1}), stochastic processes for ϵ_t and ζ_t, the surplus limit $(\bar{s})$, and the dynamic equation for the evolution of the primary surplus, equation (1), an equilibrium is values for $\{s_t, d_t, i_t, \eta_t\}$, such that expectations are rational, international creditors expect to receive i_h, equation

[7] The government's no Ponzi game constraint rules out a positive value. The no Ponzi game constraint for the household (or the aggregate of the remaining agents in the market) rules out a negative value.

(3), the primary surplus does not exceed its limit, and the government's flow budget constraint, equation (10), intertemporal budget constraint, equation (13), and expected intertemporal budget constraint, equation (14) are satisfied.

2.5.1 Dynamic Stability

Rational expectations, together with the intertemporal budget constraint, equation (13), require satisfaction of the expected intertemporal budget constraint, equation (14). This, in turn, requires that fiscal policy parameters take on values assuring that the model is dynamically stable. Explosive behavior would violate equation (14).

Taking equations (10) and (1) one period forward and taking the time t expectation yields[8]

$$E_t d_{t+1} = (1 + r_h) E_t d_t - E_t s_{t+1}, \tag{15}$$

$$E_t s_{t+1} = \gamma_h E_t d_t + \beta E_t s_t + c_h. \tag{16}$$

Equations (15) and (16) do not rule out default or default premia. Default and expectations of default can still occur, but they do not provide expected and systematic revenue or expenditure.[9] Dynamic stability requires that both eigenvalues of the matrix

$$\begin{bmatrix} 1 + r_h - \gamma_h & -\beta \\ \gamma_h & \beta \end{bmatrix}$$

be inside the unit circle. For $\gamma_h < 1$, consistent with our empirical evidence, both the determinant and trace of the matrix are positive, implying that both eigenvalues are positive. Letting θ_1 and θ_2 denote the two eigenvalues of the system, the requirement for stability is

$$(1 - \theta_1)(1 - \theta_2) > 0 \Longrightarrow \theta_1 \theta_2 - (\theta_1 + \theta_2) + 1 = \text{determinant - trace } +1 > 0$$

Substituting for values of the determinant and trace yields

$$\beta (1 + r_h) - (1 + r_h - \gamma_h + \beta) + 1 > 0,$$

requiring

$$\gamma_h > r_h (1 - \beta). \tag{17}$$

[8] Since the shocks are iid, they do not affect dynamics.

[9] $E_t \eta_{t+1} = E_t \left(\zeta_{t+1} \left[\alpha_{t+1} - E_{t+j} \alpha_{t+1} \right] \right) = E_t \left(\zeta_{t+1} \alpha_{t+1} \right) - E_t \left(\zeta_{t+1} E_t \alpha_{t+1} \right) = E_t \left(\zeta_{t+1} \alpha_{t+1} \right) - E_t \left(\zeta_{t+1} \alpha_{t+1} \right) = 0$

Stability in regime h requires the surplus responsiveness to an increase in debt (γ_h) to be larger than the interest rate (r_h), adjusted by the persistence in surplus (β). Equation (17) implies that stability requires a more aggressive responsiveness to debt in a high interest rate regime.

When the system is stable, debt and the primary surplus are expected to reach their long-run equilibrium values. We derive the long-run values by dropping time subscripts in equations (15) and (16), and solving for debt and the primary surplus as

$$d_h^* = \frac{-c_h}{\gamma_h - r_h(1-\beta)}, \qquad s_h^* = \frac{-r_h c_h}{\gamma_h - r_h(1-\beta)}. \tag{18}$$

Given the stability requirement in equation (17), a negative value for c_h yields positive long-run values of the debt (d_h^*) and the primary surplus (s_h^*). In a high interest rate regime, the long-run values in equation (18) will be higher, for given values for c_h and γ_h. However, the values for c_h and γ_h can also change in the high interest rate regime, with an increase in surplus responsiveness (γ_h) or an increase in the constant (c_h), both reducing the value for long-run debt.

We use a phase diagram to understand how the surplus and debt adjust to the long-run equilibrium following a shock. We choose parameter values for which stability holds and for which $c_h < 0$ and $r_h - \gamma_h < 0$, consistent with empirical estimates presented later. To derive the equations for the phase diagram, solve each equation, (15) and (16), for the expected future value as a function of current values, subtract the time-t values of the left-hand side variables from the expected future values in each equation, set expected changes to zero for each equation, and solve for the value of current debt as a function of the current value of the surplus to yield

$$d_{t(\Delta E_t d_{t+1}=0)} = \frac{c_h + \beta s_t}{r_h - \gamma_h}, \tag{19}$$

$$d_{t(\Delta E_t s_{t+1}=0)} = \frac{-c_h + (1-\beta)s_t}{\gamma_h}. \tag{20}$$

Figure 1 contains the phase diagram for a particular regime h with debt (d_t) on the vertical axis and (s_t) on the horizontal axis.[10] The $\Delta E_t d_{t+1} = 0$ curve plots values for d_t for each value of s_t using equation equation (19), and the $\Delta E_t s_{t+1} = 0$ curve does the same for equation (20). The $\Delta E_t d_{t+1} = 0$ curve, equation (19), has a negative slope and the $\Delta E_t s_{t+1} = 0$ curve, equation (20), has a positive slope. Equations (19) and (20)

[10] To construct Figure 1, we used $\beta = 0.65, r_h = 0.002$ and $\gamma_h = 0.02$.

divide the system into four regions, with the arrows of motion revealing the expected direction of movement of debt and surplus in each region.[11] Point E represents the long-run equilibrium values for debt and the primary surplus, given by the two equations in (18).

Adjustment paths ACE and BWLE reflect paths for expected future values of the primary surplus and debt conditional on initial values of the debt-surplus pair at A and B, respectively.[12] A series of shocks could send the system to a point like A. The adjustment path, ACE, illustrates how the system is expected to move in the absence of additional shocks. Stability assures the system returns to its long-run equilibrium, satisfying the government's actual and expected intertemporal budget constraints.

2.5.2 Boundary Locus

When there is an upper bound on the primary surplus, stability is necessary to assure that the primary surplus does not exceed the surplus limit. Without stability, the primary surplus and/or debt could become explosive, eventually exceeding any limit. However, stability is not sufficient, to assure that the primary surplus, following equations (15) and (16), is not expected to exceed the surplus limit. This is because adjustment paths to the stable equilibrium could require expected future surpluses to exceed the surplus limit along the way.

The highest possible future surpluses must be consistent with the fiscal feedback rule and with the surplus limit ($\bar{s}$). We impose a surplus limit in Figure 1, and label it $\bar{s}$. Therefore, for the country to avoid default, initial values of the debt and surplus must place the system on a path, along which no future surplus along the path exceeds $\bar{s}$. The adjustment path, labeled BWLE, which peaks at $\bar{s}$ (the value of the surplus at point L), contains the set of highest feasible future primary surpluses. When future surpluses along the adjustment path remain below $\bar{s}$ without default, the government's expected intertemporal budget constraint is satisfied without default, and the country is solvent.

We define a "boundary" locus, which separates values for debt-surplus pairs for which the government is solvent, from those for which it is not, and label it BWZ. The boundary locus has two parts: the upward sloping portion of the adjustment path given by BW,

[11] Arrows of motion are derived by taking the derivative of the equations for the expected change in the variable with respect to the current value of either debt of the surplus. For example, the derivative of $\Delta E_t d_{t+1}$ with respect to d_t is $r_h - \gamma_h$, which is negative, given our parameter values. Therefore, if the system is initially on the $\Delta E_t d_{t+1} = 0$ curve and debt falls, it is expected to rise next period, yielding the verticle upward arrows for values of debt below $\Delta E_t d_{t+1} = 0$.

[12] The adjustment paths are derived from equations (15) and (16) and their shape depends on parameter values, c_h, γ_h, r_h, β, and the value for the initial debt-surplus pair.

and the flat extension given by WZ, equivalently $\bar{d}$. If the initial debt-surplus pair is vertically below BW, then expected future surpluses will peak at a value less than $\bar{s}$, implying solvency. Additionally, if the initial debt-surplus pair is vertically below WZ, then expected future debt declines and the country can avoid breaching $\bar{s}$ by reducing its current primary surplus below WL.[13] Since surpluses along WL do not exceed $\bar{s}$, these debt-surplus pairs also yield solvency. Alternatively, for debt-surplus pairs vertically above BWZ, future surpluses exceed $\bar{s}$, implying that any paths above BWZ are infeasible. Therefore, the path BWZ separates debt-surplus pairs, which are feasible without default and those which are not.

The literature often defines the debt limit as the highest value of debt yielding solvency. In Figure 1, this is the highest value of debt along path BWLE, point W. We have labeled this value $\bar{d}$. The boundary locus becomes horizontal once debt reaches $\bar{d}$. However, given the dynamic behavior of the primary surplus, a government with debt equal to $\bar{d}$ is insolvent unless the primary surplus is at least as large as s_W (value of surplus at point W). When primary surpluses are smaller than s_W, with debt equal to $\bar{d}$, the adjustment path yields debt-surplus pairs above the boundary locus. Therefore, positions with smaller surpluses represent insolvent positions because expected future surpluses violate the surplus limit. Our definition of the debt limit is the value of debt below which the government is solvent and above which it is insolvent. Therefore, our debt limit is the value of debt along the boundary locus BMZ for a particular value of the primary surplus. Therefore, the debt limit relevant for default depends on the value of the current primary surplus.

2.6 Fiscal Space

The traditional definition of fiscal space is the difference between the largest possible value of debt and its current value. In Figure 1, the largest possible value of debt is $\bar{d}$ (the value of debt at point W), determined as the peak of the adjustment path whose primary surplus peak is the surplus limit, $\bar{s}$ (value of surplus at point L). Assume that the actual debt-surplus pair is at point A. The traditional measure of fiscal space is the debt limit, $\bar{d}$, minus the value of debt at point A. However, this traditional measure ignores dynamics. If the country received a shock to debt equal to this traditional measure of fiscal space, then the debt-surplus pair would be above the boundary locus BWZ, a position of insolvency.

[13] For the small region below WZ and above WL, the government essentially chooses to adjust the primary surplus more slowly to debt, allowing debt to fall without violating the surplus limit. Less austerity is likely to be viable politically.

The largest shock to debt that the country could receive and avoid insolvency is smaller than the vertical distance between $\bar{d}$ and point A, implying a smaller fiscal space.

The dynamic behavior of debt and the primary surplus requires an alternative measure of fiscal space. We define fiscal space as the largest one-period-ahead increase in debt, for a particular surplus, for which the country is expected to remain solvent, equivalently remain below the boundary BWZ. If the economy begins at debt-surplus pair A in Figure 1, and if expected future shocks are zero, then it is expected to transition along the adjustment path AE to point C. Fiscal space is the maximum increase in debt from point C, subject to the constraint that debt not be above BWZ, following adverse changes in the exogenous shocks (ζ_t, ϵ_t).[14]

An adverse debt shock due to growth $(\psi_t = [\zeta_t - 1]\,(1 + r_h)\, d_{t-1} > 0)$ would raise debt with an unchanged primary surplus. The vertical distance between point C and the boundary locus at point D, the length of CD, is one measure of fiscal space. Alternatively, an adverse surplus shock $(\epsilon_t < 0)$ would raise debt and reduce the primary surplus by equal amounts. The reduction in the primary surplus sends the economy horizontally from point C to point G, while the equal increase in debt moves it vertically from point G to point F. Therefore, another measure of fiscal space is the vertical length of GF. Fiscal space CD exceeds fiscal space GF because the primary surplus at point G is lower, which in turn yields a lower expected present value of future surpluses implying a lower intertemporal-budget-balancing value for debt. The shocks could occur in combination, yielding measures of fiscal space between these two distances.

Therefore, when the relevant portion of the boundary locus is upward-sloping (BW), fiscal space is not one number but a range of values, captured by the vertical distances between the FD portion of the boundary locus and GC. Therefore, fiscal space depends on the value of the primary surplus as well as the value of debt. When the relevant portion of the boundary locus is flat (WZ), the two measures of fiscal space are identical.

2.7 Default Risk

Default risk is the likelihood that exogenous shocks (ζ_t, ϵ_t) send the debt-surplus pair beyond the boundary locus, requiring default to restore equilibrium. Following Ghosh et al. (2013) and Daniel and Shiamptanis (2012), we assume that the exogenous shocks (ζ_t, ϵ_t) have a bounded, symmetric, mean-zero normal distribution, with bounds $\left(\pm\bar{\zeta}, \pm\bar{\epsilon}\right)$ given by three standard deviations. In an unbounded normal distribution, the probability

[14] Fiscal space is analogous to the distance variable in Daniel and Shiamptanis (2012).

of receiving a shock greater than three standard deviations is only 0.3%, implying that we capture virtually all the variation in the data with these bounds. We use the likelihood, that shocks send the system above the boundary locus in one period, to place the countries into three risk categories, "safe", "risky", and "highest risk".

Consider an economy in which endogenous expectations of default are zero and which is expected to reach point C from point A in the absence of exogenous shocks. Consider exogenous shocks to the debt equation first. An adverse three standard-deviation shock $(\bar{\zeta})$ to ζ_t raises ψ_t by the size of the shock adjusted for values of the interest rate and debt (equation 11). In Figure 1, the vertical distance from C to H is a measure of the magnitude of the shock. Larger values for debt and/or the interest rate imply larger values for CH. From the surplus equation (1), an adverse three standard-deviation shock to ϵ_t, denoted by $-\bar{\epsilon}$, reduces the surplus and raises debt by equal amounts. If we measure the size of the shock by CM, then the system moves from C to K, where the distance MK equals the distance CM. Analogously, adverse three standard-deviation shocks to both ζ_t and ϵ_t would send the system from point C to point J.

When the trapezoid labeled CHJK lies fully below the boundary BWZ, virtually no exogenous pair of shocks (ζ_t, ϵ_t) could push the economy over its boundary and cause default in one period. The one-period ahead risk of default is zero, and endogenous expectations of default are zero ($E_{t-1}\alpha_t = 0$). We label countries, whose trapezoid CHJK lies fully below the boundary BWZ, "safe."

However, when the trapezoid is not fully below the boundary BWZ, as in the case in Figure 1, some combinations of ζ_t and ϵ_t could exhaust fiscal space and cause default. The one-period ahead risk of default is positive. When shocks are not large enough to cause default, expectations of default are positive ($E_{t-1}\alpha_t > 0$). Expectations of default themselves imply that debt is expected to travel from the initial point A to a point above C. Therefore, expectations of default raise the debt level, thereby anchoring the trapezoid CHJK at a point above point C. When the trapezoid is anchored at a value of debt larger than C, a larger fraction of the area of the trapezoid is above the boundary BWZ. Therefore, there is a higher likelihood that exogenous adverse shocks (ζ_t, ϵ_t) would push the economy beyond the boundary locus. Default risk is increasing at an increasing rate as fiscal space shrinks, equivalently as a larger portion of the trapezoid rises above BWZ.[15] We label countries, whose trapezoid CHJK is partially above and partially below the boundary BWZ, as "risky."

[15] Using a bounded normal distribution of shocks, Daniel and Shiamptanis (2012) show that risk of default is increasing at an increasing rate as fiscal space shrinks toward zero.

For a value of the debt-surplus pair on the boundary locus BWZ, endogenous expectations of default are high causing debt, in the absence of default, to travel above the adjustment path BWZ. Fiscal space is zero and the trapezoid CHJK is fully above the boundary locus. The country cannot avoid default even if it receives the largest possible shocks. Therefore, once the economy has transited to a position on BWZ, default is necessary to restore solvency.

A country which has exhausted its fiscal space has a crisis with probability one in our model. However, our estimate of the expected future path of primary surpluses is based on historical behavior. Going forward, that behavior could change in ways our historical estimates cannot capture. Therefore, we do not claim that a country which has exhausted fiscal space faces crisis with certainty. It could avert crisis by making a historically unprecedented and credible policy change. Given the low probability of such a credible policy change, we are comfortable labeling these countries at "highest risk", but not in claiming a one-period-ahead crisis probability of unity.

We categorize crisis risk, but we cannot predict the timing of a crisis perfectly because shocks are random. Even in the case of a trapezoid fully above the boundary locus, we could predict the timing only if we were certain about the surplus limit. Since our estimate of the limit could be too low, this case indicates the riskiest situation, but does not imply a crisis probability of unity.

We do not explicitly consider serially correlation of shocks, but serial correlation could affect the position to which the system is expected to transit from A. If an adverse shock to debt was responsible for the position A, and if that shock had positive serial correlation, then the expectation of the future shock would be negative, not zero. Therefore, debt would be expected to travel to a higher position than C. Positive serial correlation would imply more risk from an adverse shock, and less risk from a favorable shock than our model implies. However, there is no systematic change in risk due to serial correlation. And for most countries, there is no empirical evidence of serial correlation in shocks.[16]

2.8 Impact of Different Interest Rate Regimes

The value of the growth-adjusted interest rate in each regime (r_h) is important in determining risk, both through its role in creating dynamic stability and through its effect on the position of the boundary locus. Figure 2 illustrates the implications of a higher r_h, keeping other parameters, including those in the fiscal feedback rule, unchanged. A higher

[16] See Appendix 8.

r_h shifts the $\Delta E_t d_{t+1} = 0$ schedule to the right and rotates it clockwise, widening the area for which debt is rising, and increasing the long-run values of debt and the surplus. Point E' represents equilibrium values for the debt and the surplus (equation 18) with a higher value for r_h.

The new boundary locus, consistent with the fiscal feedback rule and with the surplus limit $(\bar{s})$, is lower in a high interest rate regime, and it is denoted by $B'W'Z'$ in Figure 2. The new boundary contains the set of highest possible future values of debt for each value of primary surplus, with the largest possible value of debt $\left(\bar{d}'\right)$ occurring at point W', implying that a country is able to sustain lower levels of debt under a high interest rate regime.

The interest rate regime also affects the evolution of debt by changing the adjustment path from ACE to $AC'E'$. Compared with Figure 1, if the economy is expected to travel to the same surplus, represented by the surplus at points C and C', debt will be larger for the same primary surplus, anchoring the trapezoid at a higher value for debt. Additionally, the higher r_h raises the distance $C'H'$ due to the effect of the interest rate on ψ_t through the maximum shock to ζ_t in equation (11).

The effect of the higher interest rate on the boundary locus, together with its effect on the position and size of the trapezoid, increase risk. Governments could choose to offset some of that risk by choosing different parameters in the fiscal feedback rule when the interest rate is higher.

3 Empirical Results

Our sample consists of nine high-debt advanced economies, Belgium, Canada, France, Greece, Ireland, Italy, Japan, Portugal, and Spain. We choose high-debt economies, because countries with high debt are more likely to experience a solvency crisis. We choose advanced economies because the European crisis has shown that sovereign default is not confined to emerging and developing economies. We want to understand debt tolerance in advanced economies. Finally, for default to ever be necessary to restore solvency, the country must be following active-monetary passive-fiscal policy (AMPF). Many of the countries are EMU countries who do not have control over their monetary policy. With the price level determined by active monetary policy, passive fiscal policy must assure satisfaction of the intertemporal budget constraint. Unanticipated default is a source of revenue. In addition to high-debt EMU countries, we added Japan due to its very high debt level and Canada to illustrate particularly responsible fiscal policy. For these two countries, we assume that they follow AMPF and estimate their fiscal rules.

We are not claiming to estimate whether fiscal policy is active or passive. We are assuming that it is passive and that it will remain passive. Given passive fiscal policy, we are estimating the responsiveness of the surplus to lagged debt and the lagged surplus to assess the risk of default.

We use OLS to estimate fiscal feedback rules for the nine high-debt countries using annual data during the period 1980-2007. We cut our sample in 2007 for two reasons. We want estimates to construct the boundary BWZ, along which the probability of a crisis next period is zero. Prior to 2007, countries experienced neither fiscal crises nor high-debt, which could have created positive crisis expectations. Second, we want to use data prior to the global financial crisis (in-sample-data) to assess solvency risks after the global financial crisis between 2008 and 2019 (out-of-sample data).

3.1 Interest Rate Regimes

We begin by identifying different interest rate regimes in our sample, where the empirical interest rate is always the growth-adjusted interest rate. These rates have similar movements over the sample 1980-2007 for all countries.

In the early-1980's, growth-adjusted interest rates rose with a sharp increase in nominal interest rates, and they remained positive until the late-1990's for most countries. In the late 1990's, growth-adjusted interest rates declined and even turned negative for some countries. The negativity comes from the high growth rates compared to the interest rate.

To identify different interest rate regimes for each country, we test for multiple break points using the sequential procedure of Bai and Perron (1998, 2003).[17] We allow for up to 5 breaks and 15% trimming. For the majority of the countries, we find two separate interest rate regimes at the 5 percent significance level, a high interest rate regime at the beginning of the sample, and a low interest rate regime later in the sample. The dates for each regime over the sample 1980-2007, together with the mean value of the growth-adjusted interest rates (percents) in each regime, r_h, are given in Table 1. For Japan and Italy, we find one interest rate regime, and for France and Greece we find three regimes, a low interest rate regime at the beginning and the end of the sample, and a high interest rate regime in the middle of the sample. In Table 1, r_1 and r_3 represent the low interest rate regimes, and r_2 represents the high interest rate regime.

[17] We obtain almost identical results when we use the Bayesian Information Criterion and the modified Schwarz criterion of Liu et al. (1997).

We also use Hamilton's (1989) Markov switching model to obtain estimates of the probability transition matrix. We estimate a two-state Markov switching model in which the mean of the growth-adjusted interest rate is subject to regime switching. We find low probabilities of switching between the two interest rate regimes, as illustrated in Table 2.[18] Additionally, for all countries we find that the probability of the low interest rate regime broadly mimics the results of the structural break tests. In particular, for most countries the probability of the low-interest rate regime is close to 1 in the early 1980's, and later drops to zero until the late 1990's, after which it reverts to unity.

3.2 Estimates of Fiscal Feedback Rules

Next, we use OLS to estimate fiscal feedback rules for each country (equation 1), using the break dates from Table 1 to allow parameter estimates, γ_h and c_h, to differ across interest rate regimes. Results are contained in Table 3. The results imply that surplus responsiveness (γ_h) increases in interest rates (r_h), as necessary to mitigate the impact of higher interest rates on long-run values. In the high interest rate regime, *regime* 2, the surplus responsiveness (γ_2) is positive and statistically significant at the 5% percent level for all countries. In *regime* 3, when the interest rates fall for all countries, responsiveness (γ_3) falls for all countries. Similarly, for France and Greece in the early low-interest rate regime, *regime* 1, the responsiveness (γ_1) is not statistically different from zero. In the high interest rate regime, countries are responding to debt, but in the low interest rate regimes, countries are responding less or not at all. Therefore, responsiveness increases systematically with the interest rate.[19] There is no systematic change in the constant (c_h) across interest rate regimes.

For countries in which we find more than one interest rate regime, we test whether the interest rate regimes are distinct with F-tests for the equality of the regimes. Table 4 reports the p-values of the F-tests. For most of the countries, the high interest rate regime, *regime* 2, is statistically different from the late low-interest rate regime, *regime* 3,

[18] We are able to identify the probability of switching from a low interest rate regime to a high one using the beginning of the sample. The Markov switching model uses the few observations at the beginning of the sample (early 1980's) to estimate the transition probabilities from the low-interest rate to the high-interest rate. In contrast, the structural break tests (with trimming at the beginning and end of the sample period) can only detect the decline in the growth-adjusted interest rate in the late 1990's. In an earlier version of the paper with data starting in 1970, we showed that the structural break tests also detected the increase in the growth-adjusted interest rates in the early or mid-1980's.

[19] Ghosh et al. (2013) use a panel model assume that the responsiveness depends on the debt level. We provide an alternative explanation for the time-varying responsiveness to debt. We find that responsiveness not only varies across countries, but also across time based on the interest rate regime.

at the 5 percent level. For France and Greece, the early low-interest rate regime, *regime* 1, is statistically different from the highest interest rate regime, *regime* 2, and the two low-interest rate regimes are not statistically different.

Our first criterion for sustainability is that the dynamic system be stable. Our parameter estimates imply that the system is stable in each interest rate regime. Stability from equation (17) requires comparison of the responsiveness in the particular interest rate regime (γ_h) with an interest rate term in that regime ($r_h(1-\beta)$). In *regime* 2, the responsiveness is high enough to satisfy the stability requirement. In *regimes* 1 and 3, despite the fall in the responsiveness, it remains higher than the interest rate term, satisfying the stability requirement for all countries. Therefore, the fiscal parameters satisfy the stability criterion for all countries in each interest rate regime.

Next, we consider the effect of the interest rate regime on the equilibrium value for long-run debt from equation (18) for each country. We use the parameter values from Table 3 with the interest rates from Table 1. Long-run values are given in Table 5. For almost all the countries, the long-run value of debt is highest in the high interest rate regime, implying that countries do not fully adjust the parameters of the fiscal feedback rule to counter the increase in the long-run value of debt implied by a higher interest rate.

Finally, we test our assumption that the value for β is the same across interest rate regimes. When we allow the β to differ in each interest rate regime, we find that the values for β in the low and high interest rate regimes are not statistically different for all countries in our sample at the 5 percent significance level. The t-test, χ-test and F-test fail to reject the null hypothesis that the two values for β are the same during the two interest rate regimes, implying that the persistence in the primary surplus does not depend on the interest rate environment. However, at the 10% significance level, the values for β differ across interest rate regimes only for Canada. The value for β in the high interest rate regime is larger and that the values for γ_h do not differ across regimes at a 10 percent significance level. Therefore, depending on the significance level we choose, Canada could respond to the high interest rate regime by increasing the persistence of the surplus or increasing the responsiveness to debt. Either response tends to reduce the value of long-run debt (equation 18). We present results for Canada using the single value for β to be consistent with other countries' results and because the specification is accepted using a 5 percent significance level. However, results on risk assessment do not change with the alternative specification.[20]

[20] A larger β raises risk when debt is above its long-run equilibrium value and rising because it makes the slope of the adjustment path steeper. However, Canada is not in this position.

Leeper and Li (2017) raise questions about estimating the responsiveness of the primary surplus to lagged debt using surplus regressions. They illustrate the problem with a simple model in which policy parameters can take on values associated with active monetary, passive fiscal (AMPF) policy compared with the opposite, passive monetary, active fiscal (PMAF) policy. Their objective is to determine whether the primary surplus responds to lagged debt in a fiscal feedback equation which does not contain a lagged primary surplus. In this set-up a positive value for γ_h implies passive fiscal policy. They illustrate bias in the estimate of γ_h due to general equilibrium effects operating through interest rates when policy is PMAF, and due to correlation of the AR(1) surplus error ($\tilde{\epsilon}_t$) with lagged debt for both AMPF and PMAF policies. They conclude that regression inferences could be misleading. This is an extension of Cochrane's (1998) argument that empirical tests cannot distinguish between AMPF and PMAF policies.

This criticism does not apply to our surplus regressions. We use a set of countries for which we can argue that the fiscal authority has been passive. We assume that γ_h is positive and large enough to yield passive fiscal policy – we are estimating its magnitude subject to this restriction. Under AMPF, the general equilibrium effects do not create bias and the regressions yields accurate inferences. That leaves the autocorrelation between the lagged debt and the error ($\tilde{\epsilon}_t$), due to an AR(1) error in the regression as the source of potential bias. The AR(1) error is an assumption of the Leeper and Li (2017) regression, which they claim has empirical verification. In contrast to Leeper and Li (2017), our surplus regressions include a lagged dependent variable, which reduces and usually eliminates the serial correlation. When the errors are uncorrelated, the bias disappears. We test for serial correlation in $\tilde{\epsilon}_t$ using the Ljung-Box Q-statistic and Breusch-Godfrey Lagrange multiplier test. For all countries, both tests suggest that there is no first-order serial correlation in $\tilde{\epsilon}_t$. Therefore, the OLS estimates do not mislead.

4 Fiscal Space and Risk Categories

We compute out-of-sample estimates of fiscal space over the period between 2008 and 2019 for each country. Our estimates of fiscal space allow us to separate countries into risk categories and identify fiscal crises. To measure fiscal space, we need to model the environment in each country in each year, using estimates for surplus limits, exogenous shocks, boundary loci under the two different interest rate regimes, and estimates of interest rate regimes for each country after 2007.

4.1 Environment for Risk Assessment

4.1.1 Surplus Limits

We consider two approaches to determining our initial estimate for the surplus limit, $\bar{s}$, and take the maximum of the two. For the first, we follow Tanner (2013) and Collard et al (2015) and choose $\bar{s}$ to equal to the maximum historical primary surplus observed in our sample.[21] For the second, we use our estimated adjustment paths associated with every historical debt-surplus pair to estimate the maximum surplus and debt along each adjustment path. Since agents were lending at historical debt-surplus pairs, these adjustment paths must be consistent with solvency. The maximum surplus implied by this technique is the peak surplus on the highest adjustment path, point L in Figure 1. We make calculations using both approaches, and select the one which provides the largest value for $\bar{s}$. Estimates for surplus limits for each country in 2008, $\bar{s}$, are given in Table 6. For Belgium, Canada, Ireland, Italy, Japan and Spain, the historical maximum surplus is larger. For the remaining three countries, France, Greece and Portugal, the adjustment paths provide the largest surplus.

Our estimate of the surplus limit, reported in Table 6, is based on a country's historical experience with fiscal policy up through 2007. It could be an underestimate if the country is actually able to generate a surplus higher than is consistent with historical behavior. Therefore, additional information, revealed over time, could require an adjustment of the surplus limit. Specifically, when shocks send the debt-surplus pair above the initial estimated boundary locus, the country enters our highest risk category. However, if the country retains access to financial markets, then our inference is that the market believes the country can generate surpluses higher than our initial measure of $\bar{s}$. In these circumstances, we update our estimate of the surplus limit by updating the boundary locus to be the expected adjustment path from the new debt-surplus pair. The peak surplus along the new path becomes our new estimate of $\bar{s}$. We remain comfortable with our ex ante risk assessment category of "highest risk" because the country's ability to generate a surplus higher than those consistent with history is uncharted territory and, therefore, a low probability event.

[21] We justify using historical information to determine ability to pay for the same reasons that private credit markets use a household's history of borrowing and lending to set credit limits.

4.1.2 Shocks

To construct the trapezoid CHJK in Figure 1, we require estimates for the exogenous shocks (ζ_t, ϵ_t). For the surplus shock (ϵ_t), we use the standard deviation of $\epsilon_t = \delta_1 \tilde{y}_t + \delta_2 \tilde{g}_t + \tilde{\epsilon}_t$, where $\tilde{y}_t$ is the output gap, $\tilde{g}_t$ is the spending gap and $\tilde{\epsilon}_t$ is the residual of the surplus regression. Table 7 contains the standard deviations. We find that Japan has the largest standard deviation to ϵ_t of 1.895. We set the upper bound on the surplus shocks $(\bar{\epsilon})$ to three standard deviations and it is equal to the horizontal distance CM. For Japan, $\bar{\epsilon} = 5.7$, implying that an adverse three-standard-deviation surplus shock will reduce the Japanese surplus and raise the debt by 5.7% of GDP.

The values of the ζ_t shock are computed using equations (10) and (11).[22] Since they contain r_h, they differ in the low and high interest rate regimes. The standard deviations to ζ_t are reported in Table 7. Greece has the largest standard deviation to ζ_t of 0.081 in *regime* 2 and 0.058 in *regime* 3. The upper bound $(\bar{\zeta})$ is set to three standard deviations, such that $\bar{\zeta} = 0.24$ in *regime* 2 and $\bar{\zeta} = 0.18$ in *regime* 3. The Greek vertical distance CH in 2019 (ψ_{2019}), created by $\bar{\zeta}$, is 46 using r_2 or 31 using r_3, implying that an adverse three-standard deviation debt shock will increase the Greek debt by 46% of GDP in a high interest rate regime or 31% of GDP in a low interest rate regime.

4.1.3 Boundary Locus under Different Interest Rate Regimes

The position of the boundary locus depends on the interest rate regime. We derive the boundary locus both in low and high interest rate regimes. For the low interest rate regime, we use values of the estimated coefficients for the *regime* 3 fiscal feedback rule from Table 3, together with the *regime* 3 interest rates from Table 1. For the high interest rate regime, we assume that the countries returned to the high interest rate *regime* 2, and use the estimated coefficients and interest rates for *regime* 2.

In Figures 3-6, we set $\bar{s}$ equal to the maximum surplus computed in Table 6. Using estimated parameter values, we construct paths under each regime for the initial boundary locus BWZ, based on values of the debt and the primary surplus through 2007, and when needed for the latest boundary loci, based on information ending in 2019.[23] Finally, we add the adjustment path AE from the 2019 debt-surplus pair, which shows the expected path going forward. We present separate graphs for the high-interest-rate regime and for

[22] We solve for ζ_t using equations (10) and (11). During the period of 1980-2007, there were no actual or anticipated fiscal crises such that $\eta_t = 0$.

[23] The latest boundary locus in each regime is the path from the highest surplus-debt pair that the country last accessed the markets in that particular regime. We update the boundary locus in each regime using information from the particular regime.

the low-interest-rate regime for countries which had both. Table 6 reports estimates for $\bar{d}$, which is the peak debt along the initial boundary loci in Figures 3-6, and point W along the path BWLE in Figure 1.

These graphs together reveal three key takeaways. First, the highest value that debt can take on without creating a crisis, is strongly dependent on the value of the surplus. Maximum debt is given by the value of debt on the boundary locus, evaluated at the value of the surplus. For Ireland in *regime* 2, the debt values along the initial boundary locus range from 47% of GDP, when the primary surplus is low at -30% of GDP, to 119.51% of GDP when the primary surplus is higher at -0.70% of GDP, as shown in Figure 4.

Second, for all countries except Belgium, the estimated boundary loci and the debt limit $(\bar{d})$ are lower in the high interest rate regime (regime 2). Therefore, countries can experience a crisis at lower debt levels in a high-interest rate environment. Equivalently, countries can attain higher levels of debt without crisis in a low-interest rate environment. For Belgium, our results suggest that the Belgian government was able to mitigate the elevated risk of the high-interest rate environment by aggressively raising the responsiveness of the primary surplus to debt. Belgium has one of the strongest primary surplus responsiveness (γ_2) in regime 2. As a counterfactual, we use the fiscal parameters for the low-interest-rate regime (c_3, γ_3) with the high interest rate (r_2). Had Belgium kept its low-interest-rate fiscal parameters during the high interest rate regime, then $\bar{d}$ would have been 127.55%, instead of 149.20% of GDP in the high interest rate regime. Belgium offset the effect of the higher interest rate on $\bar{d}$ with a large increase in the surplus-responsiveness to debt, reducing $\bar{d}$. Other countries did not respond as aggressively to reduce risk.

Third, countries exhibit large variations in their $\bar{d}$, ranging from 73.99% of GDP for Portugal to 246.53% of GDP for Japan as shown in Table 6, yielding substantial differences in debt tolerance. Our calculations for $\bar{d}$ differ considerably from calculations based on the traditional approach, which assumes that countries immediately move to $\bar{s}$ and retain the value forever. As an example, consider Ireland, whose surplus limit in 2008 was 6.46% of GDP as shown in Table 6. Our approach yields a $\bar{d}$ of 119.51% of GDP in *regime* 2. In contrast, if we compute the debt limit in *regime* 2 using the traditional approach we obtain a much larger value of 345.45% of GDP.[24] Given the Irish experience in 2010, the value suggested by the traditional approach is unreasonable. Additionally, our results for $\bar{d}$ differ from the Ghosh et al. (2013). Their approach cannot provide debt limit estimates for Greece, Italy, Japan and Portugal, whereas for the remaining five countries

[24] According to the traditional approach, the debt limit is computed as $\frac{\bar{s}}{r_2}$.

their estimates are substantially higher than ours.[25]

4.2 Risk Assessment for Individual Countries

Prior to conducting out-of-sample risk assessment, we must determine the interest rate regime after 2007.

4.2.1 Interest Rate Regimes after 2007

We extend the data set on interest rates beyond 2007, and test for breaks in the growth-adjusted interest rates in the longer data set. For five countries, including Belgium, Canada, France, Italy and Japan, we do not find evidence of a change in the interest rate regime from the low interest rate *regime* 3 after 2007. For the remaining four countries, Greece, Ireland, Portugal and Spain, our results suggest that they moved to a high-interest rate regime in 2008, and then returned to a low-interest rate regime in the mid-2010's.[26]

We use the estimates of the surplus limit, the regime-specific coefficients in the fiscal feedback rule, the high and low interest rate regimes to place countries in risk categories in each year, identify crises which did occur, and identify potential fiscal crisis in the near future.

4.2.2 Belgium and Canada

Belgium and Canada are two high-debt countries which remained in the low interest rate regime after 2007 and did not experience debt crises over the period between 2008 and 2019. Our estimates place both countries in the "safe" category over the entire period, regardless of the interest rate regime. Figure 3 contains the boundary locus for Belgium and Canada in regime 2 (high interest) and 3 (low interest), their debt-surplus pairs between 2008 and 2019, and their trapezoids for the adjustment path from the 2019 debt-surplus pair.[27] The trapezoids reveal the potential positions of the debt-surplus pairs in 2020 conditional on receiving adverse shocks within three standard deviations. Since the

[25] Ghosh et al. (2013) debt limits using the projected market interest rates are 168.4% for Belgium, 181.1% for Canada, 176.1% for France, 149.7% for Ireland and 153.9% for Spain.

[26] The increase in the growth-adjusted interest rates is driven by the reduction in growth rates, and not by the introduction of default premia. We obtain almost identical results when we use the German or the US nominal interest rates as the risk free rates and adjust them for the country specific growth rates. We also get the same results when we calculate growth adjusted interest rates as the German rate plus the average interest rate premium over 1999-2007 adjusted for country specific growth rates.

[27] The trapezoids are anchored at the point at which the debt is expected to travel in the absence of endogenous expectations of default.

trapezoids lie fully below their boundary loci in both interest rate regimes, three standard deviations shocks to both debt and the surplus would not breach the boundary locus, implying that both countries are in the "safe" category. The probability of a one-period-ahead crisis is zero.

4.2.3 Greece, Portugal, and Ireland

Consider the opposite case, Greece, Portugal and Ireland, which all experienced fiscal crises, but were able to reaccess the markets later at higher debt levels. Figure 4 contains their debt-surplus pairs between 2008 and 2019, two boundary loci, the initial, based on the behavior up until 2007, and the latest, based on information ending in 2019, and the trapezoids from the 2019 debt-surplus pair. The debt tolerance for these countries varies considerably with the interest rate regime. The boundary loci in the high interest rate regime 2 are substantially lower than those in the low interest rate regime 3. This implies that a crisis can occur at lower debt level when interest rates are high. Our break tests imply that Greece, Portugal and Ireland returned to a high-interest-rate regime in 2008. The high interest rate environment reduced their boundary loci and triggered the fiscal crisis that they experienced.

For Portugal, shocks together with default expectations, sent the 2008 debt-surplus pair above their initial high-interest-rate boundary loci (regime 2), placing the country in the "highest risk" category. Portugal retained access to financial markets, implying that the market expected them to attain surpluses higher than historically observed. We infer that our estimate of $\bar{s}$ is too low and update the boundary locus to be the expected adjustment path from the 2008 debt-surplus pair. Going forward in time, we continue to update the boundary loci and values for $\bar{s}$ as long as the country retains access to the financial markets.[28] We retain the risk category "highest risk" because our assessment is that the ability to generate a surplus higher than historically indicated is uncharted territory and therefore a low probability event. Portugal continued to access the markets until early 2011.[29] After a couple of years in the "highest risk" category, Portugal succumbed to fiscal crisis.

For Greece, shocks together with default expectations, sent the 2009 debt-surplus pair above their initial high-interest-rate boundary loci (regime 2), placing the country in the "highest risk" category. Since Greece retained access to financial markets, we update the boundary locus to be the expected adjustment path from the 2009 debt-surplus pair.

[28] Intermediate boundary loci are not drawn to avoid clutter.

[29] The latest boundary locus in regime 2 for Portugal is the path from the 2010 debt-surplus pair.

This is the latest boundary locus in regime 2 as Greece lost access to the markets in early 2010. Similar to Portugal, after a couple of years in the "highest risk" category, Greece experienced a fiscal crisis.

For Ireland, the 2010 shocks and default expectations sent the economy above the initial boundary locus in regime 2, placing the country in the "highest risk" category. Ireland lost access to the markets at the end of 2010. Unlike Greece and Portugal, Ireland experienced a crisis in the first period it entered the "highest risk" category. From Table 6, Ireland has the largest $\bar{s}$ in the sample (6.46% of GDP), and events are consistent with this historical being the actual maximum. The latest boundary locus in regime 2 is only sightly above the initial boundary locus.

Ireland and Portugal returned to a low interest rate environment (regime 3) in 2014, and Greece returned in 2018. In the low interest rate regime, boundary loci are substantially higher. Ireland was able to re-access the markets at the end of 2013, and Portugal regained access in mid-2014, both at higher debt levels. For both countries, all the debt-surplus pairs between 2014 and 2019 are sufficiently below the low-interest-rate boundary loci that their associated trapezoids lie fully below. With the return of the low-interest-rate regime in 2014, both countries moved back to the "safe" category with large fiscal space. In contrast, Greece remained in the "highest risk" category, even with the return of the low interest rate regime in 2018, because the 2018 debt-surplus pair is above the initial boundary locus in regime 3. When the debt-surplus pair is above the prevailing boundary locus and the country retains access to markets, we update the boundary locus to put the current debt-surplus pair on the locus. But this places the country at "highest risk" because it must be able to raise future surpluses higher than historically possible. In 2019, Greece experiences positive fiscal space as the 2019 debt-surplus pair falls below the latest boundary locus. But the fiscal space remains too small to eliminate risk because its 2019 trapezoid crosses the latest boundary locus, thereby placing the country in the "risky" category.[30] The expectations of default are positive pushing the economy closer to its latest boundary locus than implied by the 2019 adjustment path shown in Figure 3.

Our model implies that if Portugal and Ireland had remained in the low interest rate regime between 2008 and 2014, they would have avoided the "highest risk" category. All their trapezoids associated with the debt-surplus pairs between 2008 and 2014 lie fully below the initial low-interest-rate boundary locus. In contrast, Greece would have been in

[30] Our model does not include the possibility of official international transfers, but these could be especially important in allowing Greece to avoid default going forward.

the "highest risk" category between 2013 and 2018 regardless of the interest rate regime. Going forward from 2019, Ireland is "safe" in both regimes since their trapezoids in each regime both lie below the latest boundary loci. Our results place Greece in the "highest risk" category in regime 2 because its 2019 debt-surplus pair is above the latest high-interest-rate boundary locus, but in the "risky" category in regime 3. Portugal is in the "highest risk" category in a high interest rate regime 2 since its debt-surplus pair lies above the latest boundary locus and is "safe" in the low-interest rate regime since its trapezoid lies fully below the boundary locus. Therefore, risk assessment for Greece and Portugal depends importantly on the possibility of regime change going forward. Table 2 shows that, for Greece and Portugal, the probability of switching to a high interest rate is low.

4.2.4 France and Spain

Figure 5 contains the boundary loci for France and Spain under both regimes, their debt-surplus pairs from 2008 through 2019, and the trapezoids and adjustment paths from the 2019 debt-surplus pair. France remained in the low interest rate regime 3 throughout the period, with trapezoids lying fully below the boundary, placing the country in the "safe" category. However, this risk assessment must be tempered because it is conditional on France remaining in the low-interest-rate regime. Had France moved to the high interest rate regime, it would have entered the "highest risk" category in every year after 2008 since all debt-surplus pairs lie above the high-interest-rate boundary locus. And going forward from 2019, France is "safe", conditional on remaining in the low-interest-rate regime, but at "highest risk", conditional on a switch to the high-interest-rate regime.

Spain moved to the high interest rate regime 2 in 2008. During the period 2008-2012, fiscal space shrinks for Spain, as the country moves from "safe" to "risky", and finally to "highest risk" in 2012 with the debt-surplus pair on the initial boundary locus. Spain, facing high default risk premium on borrowing, received an official loan at the end of 2012 without actually losing access to markets. The official loan indicates fiscal distress, consistent with our risk assessment of "highest risk."

Our break tests imply that Spain returned to a low interest rate regime in 2014. The trapezoid associated with the 2014 debt-surplus pair (not shown) lies fully below the boundary locus in the low interest rate regime 3, placing Spain in the "safe" category. Under the low-interest-regime, Spain is able to re-access the markets at higher debt levels. Going forward from 2019, risk assessment is dependent upon the interest rate regime. If Spain remains in the low-interest-rate regime, then Spain is "safe", since its 2019 trapezoid lies fully below the boundary locus. However, if the regime switches, the 2019 debt-surplus pair is on the latest boundary locus for the high interest rate regime 2, placing Spain in the "highest risk" category.

4.2.5 Italy and Japan

Italy and Japan are two highly-indebted countries, both of which only have one interest rate regime, and both of which did not experience debt crises over the period. Comparing cross-country interest rates in Table 1, the Italian and Japanese interest rates are among the lowest in regime 2. Their boundary paths are drawn in Figure 6. Both countries experienced falling fiscal space over the period between 2008 and 2019, moving from the "safe" category, in which trapezoids (not shown) were fully below the boundary locus to the "risky" category. The trapezoids from the 2019 debt-surplus pair intersect the boundary locus. Therefore, both countries are in the "risky" category going forward from 2019. The expectations of default are positive, pushing the economy closer to the boundary loci than the adjustment paths in Figure 6 show.[31]

We consider two counterfactual experiments. In the first, we double their interest rates.[32] In the second, we reduce their surplus limits to the Portuguese one ($\bar{s} = 3.24$), which is the lowest in our sample. Had the interest rates in Japan and Italy been twice as high as the ones reported in Table 1, then Japan and Italy would have entered the "highest risk" category in 2012 and 2013, respectively. Also, had the surplus limits been lower in both countries, then Italy would have entered the "highest risk" category in 2009. Although the lower surplus limit lowers the Japanese boundary locus, it does not push the country into the "highest risk" category. Both experiments highlight the importance of the interest rate and the surplus limit in determining potential fiscal crises.

5 Conclusion

How do we determine whether or not a country has taken on so much debt that it risks a fiscal solvency crisis? Every country faces limits on the magnitude of the primary surpluses it can raise over time and, therefore, on the value of the debt it can sustain. And these fiscal limits differ across countries and across time. Belgium and Greece illustrate differences across countries as Greece suffered a crisis at a value of debt/GDP which

[31] The expected adjustment paths from the 2019 debt-surplus pair are drawn in the absence of expectations of default. When expectations of default are included, fiscal risk rises because a larger portion of the trapezoid lies above the boundary locus.

[32] The Japanese interest rate is doubled to 0.97% and the Italian one is doubled to 3.76%.

Belgium successfully managed. Several European countries regained access to markets at debt levels higher than those at which they lost access, illustrating differences across time. We propose and implement a new and simple data-based method for estimating debt limits and fiscal space, and for separating countries into risk categories. The time variation in fiscal limits is due to changes in the interest rate regime over time. The country variation is due to different surplus limits and different parameters in country fiscal feedback rules, particularly the surplus responsiveness to debt. And the initial value of the surplus contributes to time and country variation by anchoring the path of expected future surpluses for given debt.

Our methodology for identifying risk is data-driven and relies on historical data in two ways. First, we need estimates of surplus limits. We begin with the largest historical value of the primary surplus. Second, since a government facing solvency problems does not move its primary surplus immediately to its limit and retain this value indefinitely, we need a way to predict expected future values for the surplus. We estimate a fiscal feedback rule where the surplus responds to its own lag and to lagged debt, and when the coefficients determining responses can differ, depending on the interest rate regime. We find that the primary surplus response to debt is higher when the interest rate is higher. Given a debt-surplus pair and an interest rate regime, the fiscal feedback rule provides predictions for future surpluses, conditional on remaining in the same interest-rate regime. The flow government budget constraint, together with with a value for the growth-adjusted interest rate, yields predictions of future debt. We use the estimated fiscal rules to construct adjustment paths, which provide an alternative estimate for the surplus limit, as the peak of debt along adjustment paths beginning at historical debt-surplus pairs. The surplus limit we use is the maximum of the historical value and this estimated value. The peak of the debt along this path is debt limit. Surplus and debt limits differ substantially across countries and across the two interest rate regimes, yielding different tolerance for debt.

To measure fiscal space and assess risk, we use the adjustment path, with its peak at the surplus limit, to define a boundary locus above which the country is insolvent, conditional on a particular interest-rate regime. For a debt-surplus pair above the boundary locus, expected future values of the debt and/or surplus are expected to violate fiscal limits, an impossibility. We define fiscal space as the magnitude of the increase in debt, following an adverse shock, which would send the debt-surplus pair to the boundary locus. Since countries face different fiscal limits, fiscal space is not necessarily lowest for the highest debt countries. We use our measures of fiscal space to separate countries into three risk categories. The "highest risk" category contains countries which have exhausted their fiscal space. This is uncharted territory – to retain solvency without default, they must be expected to raise surpluses above values deemed feasible by historical experience.

These risk categories are conditional on a country remaining in the same interest rate regime. Using Hamilton's (1989) regime switching model, our estimates reveal a small probability of switching interest rate regimes. Therefore, to assess risk, we must make the assessment in both high and low interest rate regimes. A country which is in the "safe" category in a low-interest-rate can be in the "highest risk" category in the high-interest-rate regime.

We use the model to predict fiscal crises following the 2008 global financial crisis. Most countries were in a low-interest-rate regime. Belgium and Canada were in the "safe" category and would have remained in that category even with a regime switch. France was in the "safe" category, but would have moved to "highest risk" in the event of a regime switch. Greece, Portugal, and Ireland returned to the high-interest-rate regime in which their fiscal space was exhausted, placing them in the "highest risk" category. Ireland's crisis arrived the first year the country was rated "highest risk", and crises for Greece and Portugal arrived after a couple of years in the "highest risk" category. Therefore, for the extreme cases, countries which did experience crises and those which showed no risk, the model successfully predicts outcomes. And the result that most countries exhibited risk but did not experience a crisis, is also consistent with the model, although not a direct validation. The model also explains why crisis countries were able to exit the "highest risk" category and reaccess markets after they transited back to low-interest-rate regimes.

Going forward, risk assessment is highly dependent on the evolution of the interest rate regime. Only Belgium and Canada are perfectly "safe", even if they transit back to a high interest rate regime. All other countries except Greece remain "safe" in the low interest rate regime, but risk rises if they transit to a high interest rate, with Spain, France, and Portugal moving to the "highest risk" category.

References

[1] Arellano, C., Y. Bai, and G. Mihalache (2020) "Deadly debt crises: Covid-19 in emerging markets," Federal Reserve Bank of Minneapolis Staff Paper 603

[2] Bai, J., and P. Perron (1998) "Estimating and testing linear models with multiple structural changes," *Econometrica* 66(1), 47-78

[3] — (2003) "Computation and analysis of multiple structural change models," *Journal of Applied Econometrics* 18(1), 1-22

[4] Bi, H. (2012) "Sovereign default, risk premia, fiscal limits, and fiscal policy," *European Economic Review* 56(3), 389-410

[5] Bi, H., E. Leeper, and C. Leith (2013) "Uncertain fiscal consolidations," *Economic Journal* 123(566), F31-F63

[6] Bohn, H. (1998) "The behavior of U.S. public debt and deficits," *Quarterly Journal of Economics* 113(3), 949-63

[7] — (2007) "Are stationarity and cointegration restrictions really necessary for intertemporal budget constraints?," *Journal of Monetary Economics* 54(7), 1837-47

[8] Celasun, I., X. Debrun, and J. D. Ostry (2007) "Primary surplus behavior and risks to fiscal sustainability in emerging market countries: A "fan-chart" approach," *IMF Staff Papers* 53(3), 401-25

[9] Cochrane, J. H. (1998) "A Frictionless view of U.S. inflation," *NBER Macroeconomics Annual* 13, 323-421

[10] Collard, F., M. Habib, and J. Rochet (2015) "Sovereign debt sustainability in advanced economies," *Journal of the European Economic Association* 13(3), 381-420

[11] Daniel, B. (2014) "A graceful return of the drachma," *European Economic Review* 71, 228-43

[12] Daniel, B., and C. Shiamptanis (2012) "Fiscal risk in a monetary union," *European Economic Review* 56(6), 1289-309

[13] Daniel, B., and C. Shiamptanis (2013) "Pushing the limit? Fiscal policy in the European monetary union," *Journal of Economic Dynamics and Control* 37 (11), 2307-21

[14] Davig, T., E. Leeper, and T. B. Walker (2011) "Inflation and the fiscal limit," *European Economic Review* 55(1), 31-47

[15] Ghosh, A. R., J. I. Kim, E. G. Mendoza, J. D. Ostry, and M. S. Qureshi (2013) "Fiscal fatigue, fiscal space, and debt sustainability in advanced economies," *Economic Journal* 123(566), F4-F13

[16] Hamilton, J. D. (1989) "A new approach to the economic analysis of nonstationary time series and the business cycle," *Econometrica* 57(2), 357-84

[17] IMF (2018) "Assessing fiscal space: An update and stocktaking," IMF policy paper

[18] Leeper, E. (1991) "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics* 27(1), 129-47

[19] Leeper, E. (2010) "Monetary science, fiscal alchemy," Proceedings - Economic Policy Symposium - Jackson Hole, Federal Reserve Bank of Kansas City, 361-434

[20] Leeper, E., and B. Li (2017) "Surplus-debt regressions," *Economics Letters*, 151, 10-15

[21] Liu, J., S. Wu, and J. V. Zidek (1997) "On segmented multivariate regression," *Statistica Sinica* 7, 497–525

[22] Mendoza, E. G., and J. D. Ostry (2008) "International evidence on fiscal solvency: Is fiscal policy "responsible"?," *Journal of Monetary Economics* 55(6), 1081-93

[23] Tanner, E. (2013) "Fiscal sustainability: A 21st century guide for the perplexed," *IMF Working Paper* No. 13/89

6 Tables

Table 1 Interest Rate Regimes

	regime 1		*regime* 2		*regime* 3	
	Period	r_1 (%)	Period	r_2 (%)	Period	r_3 (%)
Belgium			1980-1996	4.00	1997-2007	0.42
Canada			1980-1998	3.40	1999-2007	-0.83
France	1980-1989	1.04	1990-1997	4.58	1998-2007	0.39
Greece	1980-1991	-3.13	1992-2000	4.91	2001-2007	-2.91
Ireland			1980-1994	1.87	1995-2007	-5.35
Italy			1980-2007	1.88		
Japan			1980-2007	0.48		
Portugal			1980-1996	3.06	1997-2007	-0.67
Spain			1980-1997	2.46	1998-2007	-2.65

Table 2 Transition Probabilities for Two Interest Rate Regimes

	Belgium	Canada	France	Greece	Ireland	Portugal	Spain
prob(High | Low)	0.14	0.10	0.05	0.05	0.05	0.04	0.05
prob(Low | High)	0.19	0.12	0.18	0.16	0.11	0.33	0.04

Table 3 Fiscal Rules with Interest Rate Dependency

	Belgium	Canada	France	Greece	Ireland	Italy	Japan	Portugal	Spain
c_1			-1.937†	-1.803					
			(0.932)	(1.086)					
c_2	-12.209‡	-5.905‡	-3.940‡	-9.282‡	-6.056‡	-6.621‡	-3.115‡	-6.415†	-4.691‡
	(1.441)	(0.721)	(1.055)	(2.351)	(1.600)	(1.899)	(0.718)	(2.790)	(1.143)
c_3	-4.578‡	-2.713*	-2.375	-9.320	-0.470			-4.301†	-0.479
	(1.145)	(1.366)	(4.812)	(7.834)	(0.524)			(1.707)	(1.272)
γ_1			0.052	0.002					
			(0.031)	(0.026)					
γ_2	0.111‡	0.072‡	0.073‡	0.113‡	0.074‡	0.072‡	0.016†	0.123†	0.077‡
	(0.012)	(0.007)	(0.020)	(0.023)	(0.018)	(0.021)	(0.006)	(0.053)	(0.017)
γ_3	0.071‡	0.039†	0.034	0.086	0.024*			0.043	0.023
	(0.012)	(0.017)	(0.075)	(0.078)	(0.013)			(0.029)	(0.023)
β	0.302‡	0.576‡	0.469‡	0.287†	0.714‡	0.494‡	0.795‡	0.215	0.357†
	(0.055)	(0.068)	(0.146)	(0.133)	(0.096)	(0.162)	(0.059)	(0.135)	(0.165)
δ_1	0.194*	0.398‡	0.167	0.043	0.067	0.078	0.380‡	0.449‡	0.308‡
	(0.099)	(0.054)	(0.124)	(0.231)	(0.107)	(0.147)	(0.072)	(0.136)	(0.087)
δ_2	-0.373‡	-0.366‡	-0.636‡	-0.198†	-0.630*	-0.146	-0.320‡	-0.460‡	-0.129*
	(0.040)	(0.046)	(0.215)	(0.090)	(0.147)	(0.119)	(0.027)	(0.126)	(0.077)
$\bar{R}^2$	0.981	0.969	0.661	0.776	0.921	0.882	0.948	0.519	0.935
SER	0.534	0.645	0.669	1.035	0.963	1.131	0.745	1.222	0.774

Note: The $*$, † and ‡ indicate statistical significance at the 10, 5 and 1 percent level, respectively.

Table 4 F-tests on Regime Equality

	Belgium	Canada	France	Greece	Ireland	Portugal	Spain
p-value($regime1$=$regime2$)	-	-	0.048	0.054	-	-	-
p-value($regime2$=$regime3$)	0.000	0.091	0.156	0.001	0.004	0.003	0.074
p-value($regime1$=$regime3$)	-	-	0.355	0.456	-	-	-

Table 5 Long Run Debt (% of GDP)

	Belgium	Canada	France	Greece	Ireland	Italy	Japan	Portugal	Spain
$regime1$	-	-	41.28	74.43	-	-	-	-	-
$regime2$	146.35	102.33	80.86	118.62	88.83	105.55	205.91	64.54	75.88
$regime3$	67.10	64.29	73.27	86.94	11.93	-	-	88.54	11.94

Table 6 Fiscal Limits (% of GDP) in 2008

	Belgium	Canada	France	Greece	Ireland	Italy	Japan	Portugal	Spain
$\bar{s}$	6.24	5.76	3.71	5.82	6.46	5.66	3.27	3.24	3.38
$\bar{d}$ in $regime2$	149.20	119.87	80.86	118.62	119.51	138.34	246.53	73.99	90.45
$\bar{d}$ in $regime3$	135.52	154.92	134.97	174.10	157.11	-	-	165.79	133.12

Table 7 Standard Deviation of Shocks

	Belgium	Canada	France	Greece	Ireland	Italy	Japan	Portugal	Spain
σ_{ϵ_t}	1.279	1.272	0.763	1.041	1.16	1.092	1.895	1.346	1.056
σ_{ζ_t} in $regime2$	0.019	0.041	0.027	0.081	0.044	0.038	0.038	0.070	0.069
σ_{ζ_t} in $regime3$	0.016	0.028	0.022	0.058	0.051			0.043	0.014

7 Appendix: Data

Variable	Description	Source
s_t	primary surplus as a percentage of GDP	OECD and AMECO.
d_t	gross government debt as a percentage of GDP[33]	OECD and AMECO.
$\tilde{y}_t$	cyclical component of the log real GDP obtained from the Hodrick-Prescott filter	OECD, AMECO and Authors' calculations.
$\tilde{g}_t$	cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter	OECD, AMECO and Authors' calculations.
ρ_t	nominal GDP growth rate	OECD and AMECO.
i_t	nominal interest rate on government bonds	OECD and AMECO.
r_t	growth-adjusted interest rate is as $r_t = \frac{1+i_{t-1}}{1+\rho_t} - 1$	Authors' calculations

The sample is from 1980-2019. We use 1980-2007 to estimate the fiscal feedback rules (in sample estima-

[33] We use gross debt as in Bohn (1998, 2007), Mendoza and Ostry (2008) and Ghosh et al. (2013). Net debt is not available for all the countries in our sample.

tion), and we use 2008-2019 for the assessment of risk (out of sample).

Summary Statistics (1980-2007)

	Debt as % of GDP			Surplus as % of GDP		
	Min	Mean	Max	Min	Mean	Max
Belgium	74.60	112.70	135.16	-8.90	2.06	6.24
Canada	45.58	78.96	108.78	-6.24	-0.28	5.76
France	21.02	46.48	67.38	-3.69	-0.79	1.24
Greece	21.41	74.09	106.85	-3.82	-0.84	3.06
Ireland	23.62	68.19	109.39	-5.69	1.50	6.46
Italy	54.39	95.31	116.89	-6.20	0.03	5.66
Japan	48.75	97.98	157.90	-8.90	-2.27	3.27
Portugal	29.09	54.73	73.68	-4.64	-1.13	2.11
Spain	15.99	44.87	65.41	-6.69	-1.31	3.38

8 Appendix: Serial correlation

In the baseline model, we assumed that the shocks ϵ_t and ζ_t are iid. In this section, we consider the implications when the shocks follow a first-order autoregressive process, AR(1) for short, given by

$$\begin{aligned} \epsilon_t &= \rho_\epsilon \epsilon_{t-1} + u_{\epsilon_t}, \qquad u_{\epsilon_t} \sim N\left(0, \sigma^2_{u_\epsilon}\right) \\ \zeta_t &= \rho_\zeta \zeta_{t-1} + u_{\zeta_t} \qquad u_{\zeta_t} \sim N\left(0, \sigma^2_{u_\zeta}\right) \end{aligned}$$

We first test empirically for the presence of an AR(1) process in the errors. Although our results suggest that for all countries there is no first-order serial correlation in $\tilde{\epsilon}_t$, which is the residual in the surplus regression and represents one of three primary surplus shock, we test for serial correlation in $\epsilon_t = \delta_1 \tilde{y}_t + \delta_2 \tilde{g}_t + \tilde{\epsilon}_t$, which includes all three types of shocks to the primary surplus, the output-gap shock ($\tilde{y}_t$), the temporary government spending shock ($\tilde{g}_t$), and the aggregate of all other shocks ($\tilde{\epsilon}_t$). Using the Ljung-Box Q-statistic and Breusch-Godfrey Lagrange multiplier test, we find evidence of first-order serial correlation in ϵ_t for Belgium, Canada and Spain. Table 8 reports the estimate of ρ_ϵ. Then we test for serial correlation in ζ_t and find evidence of serial correlation for Italy, Japan and Spain.

Next, we consider the implications of serial correlation on the theoretical model. We use Figure 1 to illustrate the effect of serial correlation on the adjustment paths, the trapezoid and default risk. In the presence of serial correlation, the expected future movements depend on the most recent shocks. Suppose that the economy starts at point A and in the previous period it received adverse surplus and debt shocks. If the shocks have a positive serial correlation, then in the next period the economy is also expected to receive adverse shocks. Therefore, the economy does not travel to point C. Instead, the economy is expected to be at a point above M, which is northwest of point C. At a point above M, the primary surplus is lower and debt is higher than point C. The higher debt stretches the CH distance and enlarges the size of the trapezoid CHJK. A larger area of the trapezoid will be above the boundary locus BWZ, raising the likelihood that the shocks together with default premia will push the economy above its boundary locus. Therefore, a positive serial correlation increases the risk of default from adverse shocks and decreases the risk from a favorable shock. The opposite also holds true. A negative serial correlation decreases the risk of default from an adverse shock and increases the risk from a favorable shock.

Table 8: Serial Correlation Tests

	Belgium	Canada	France	Greece	Ireland	Italy	Japan	Portugal	Spain
$\tilde{\epsilon}_t$	no	no	no	no	no	no	no	no	no
ϵ_t	yes (-0.336)	yes (0.543)	no	no	no	no	no	no	yes (0.503)
ζ_t	no	no	no	no	no	yes (0.613)	yes (0.553)	no	yes (0.415)

Figure 1: Fiscal space

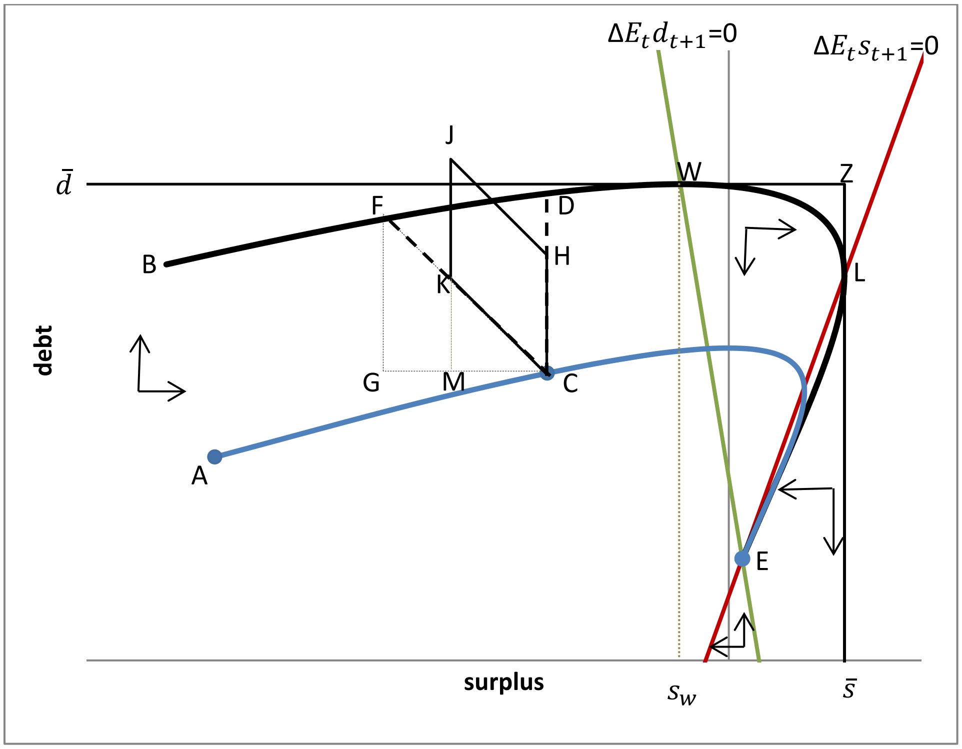


Figure 2: The implications of a high-interest rate regime. Low (solid lines) vs High (dotted lines) interest rate regimes

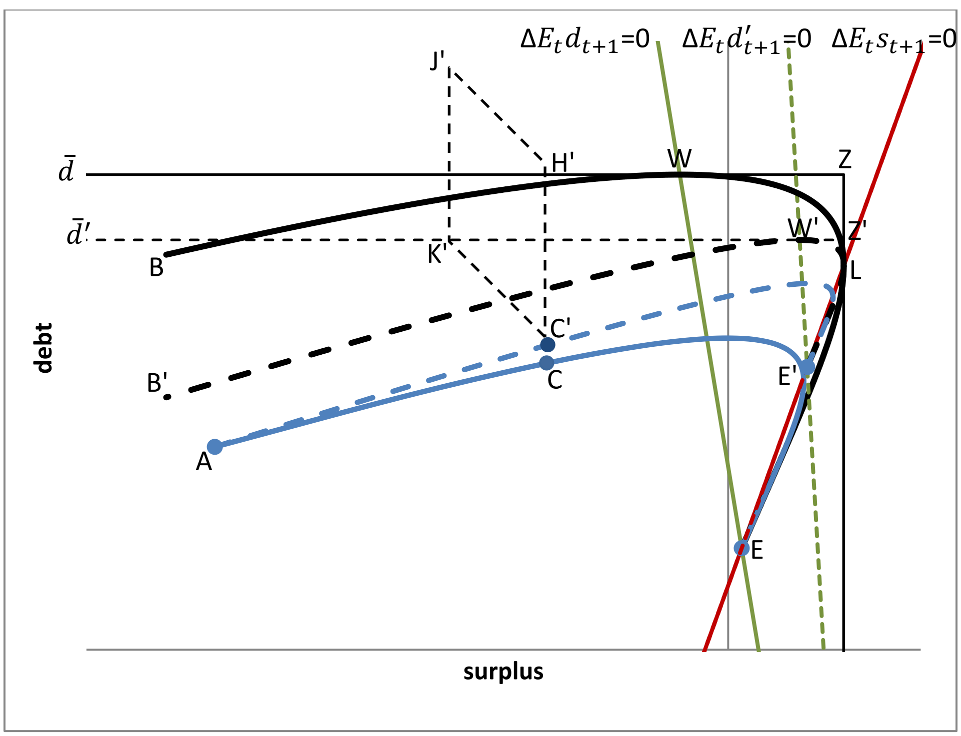


Figure 3: Belgium and Canada in the high (regime 2) and low (regime 3) interest rate regimes

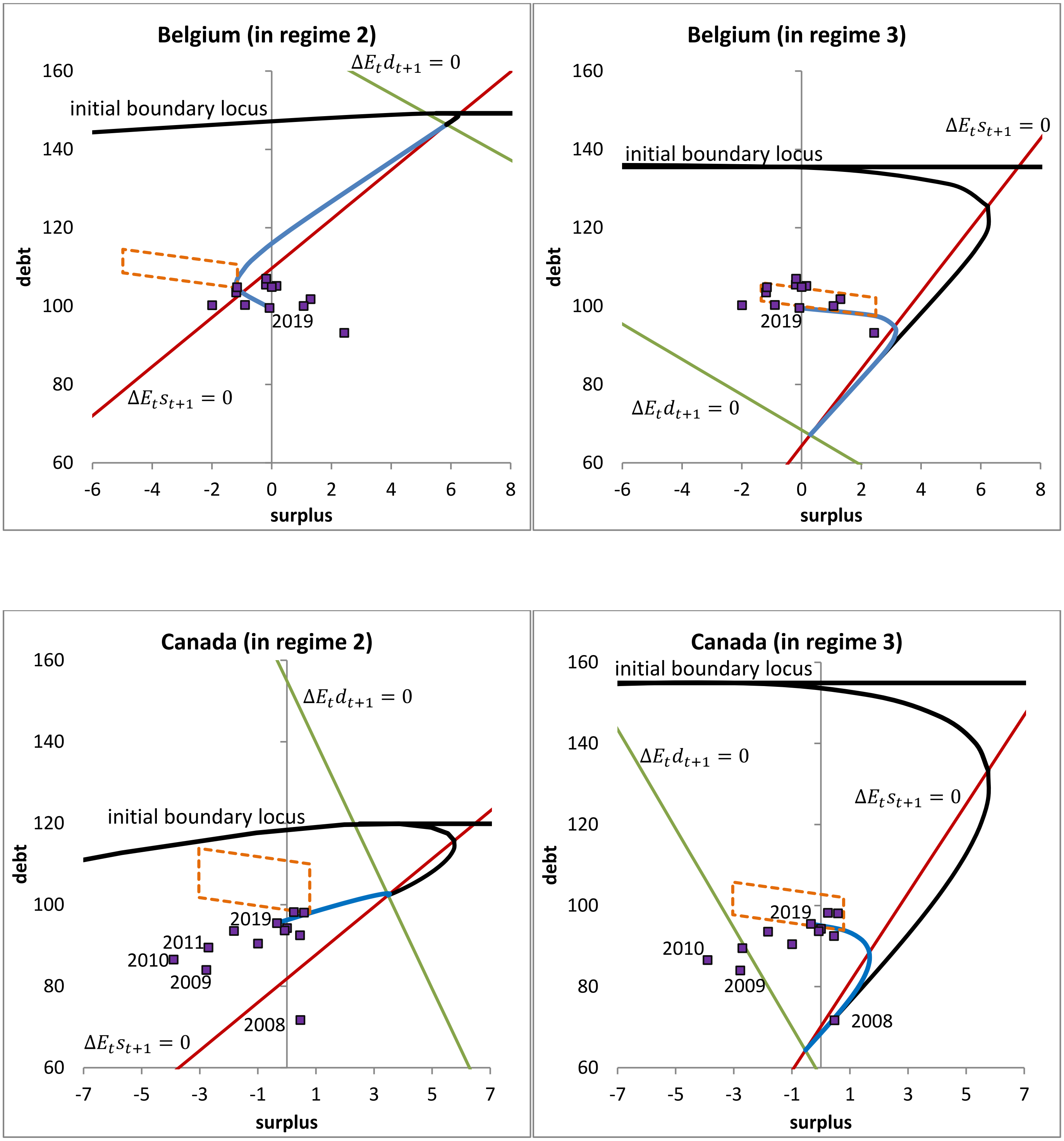
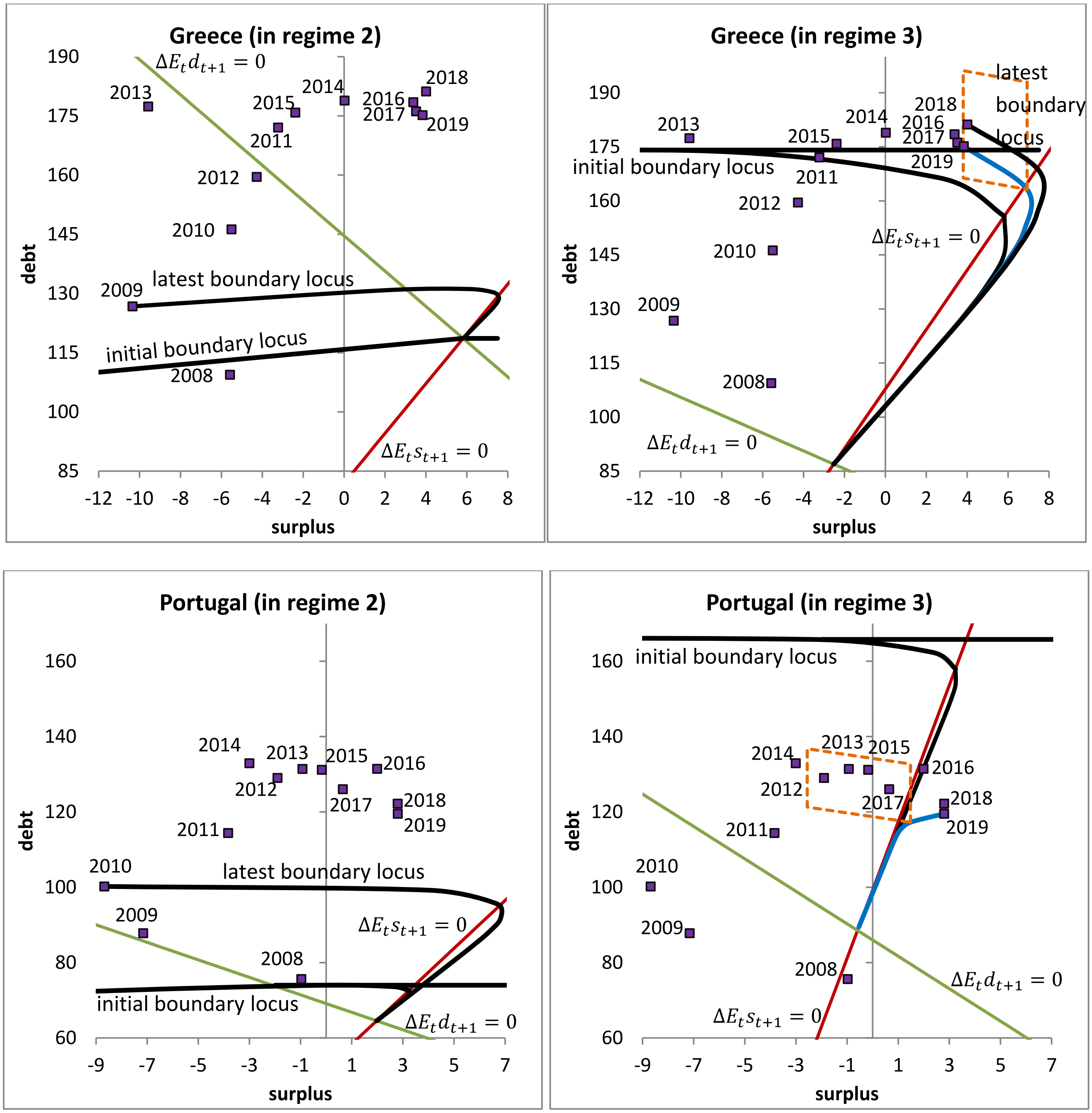


Figure 4: Greece, Portugal and Ireland in the high (regime 2) and low (regime 3) interest rate regimes



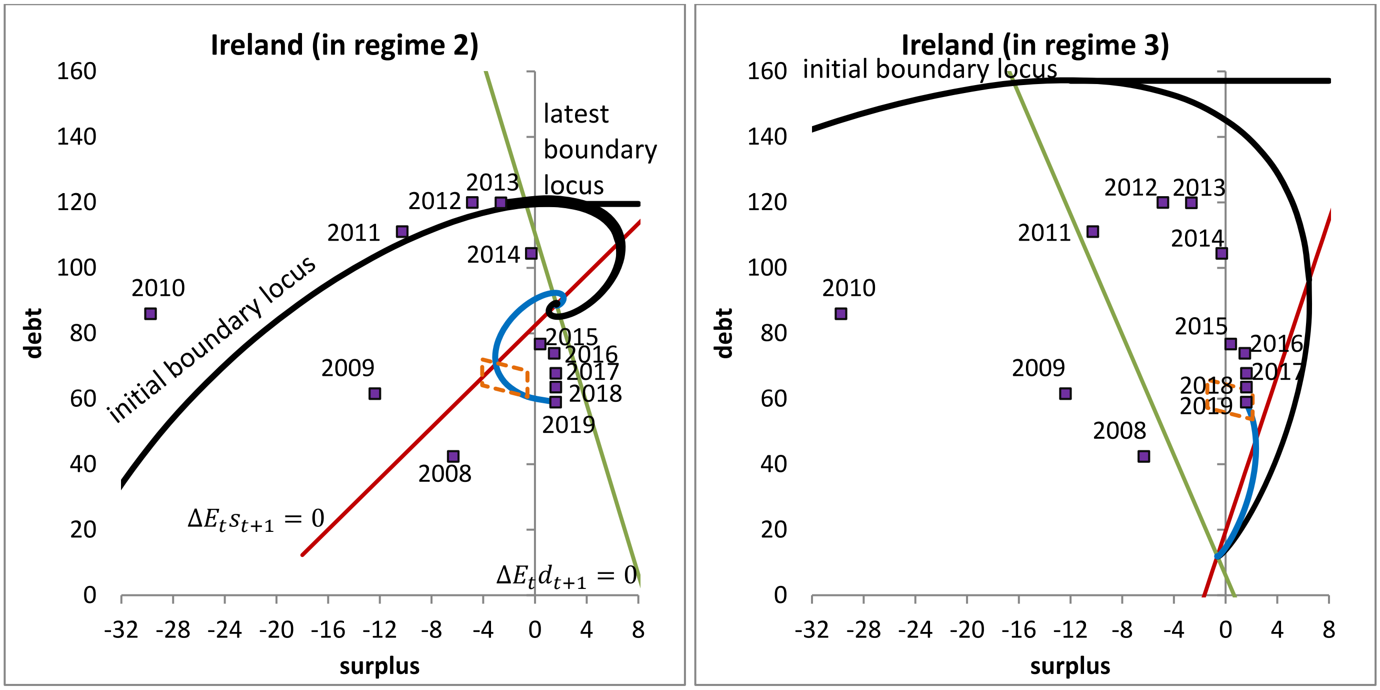

Ireland (in regime 2)
debt
surplus
latest boundary locus
initial boundary locus
$\Delta E_t s_{t+1} = 0$
$\Delta E_t d_{t+1} = 0$
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
Ireland (in regime 3)
initial boundary locus
debt
surplus
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019

Figure 5: France and Spain in the high (regime 2) and low (regime 3) interest rate regimes

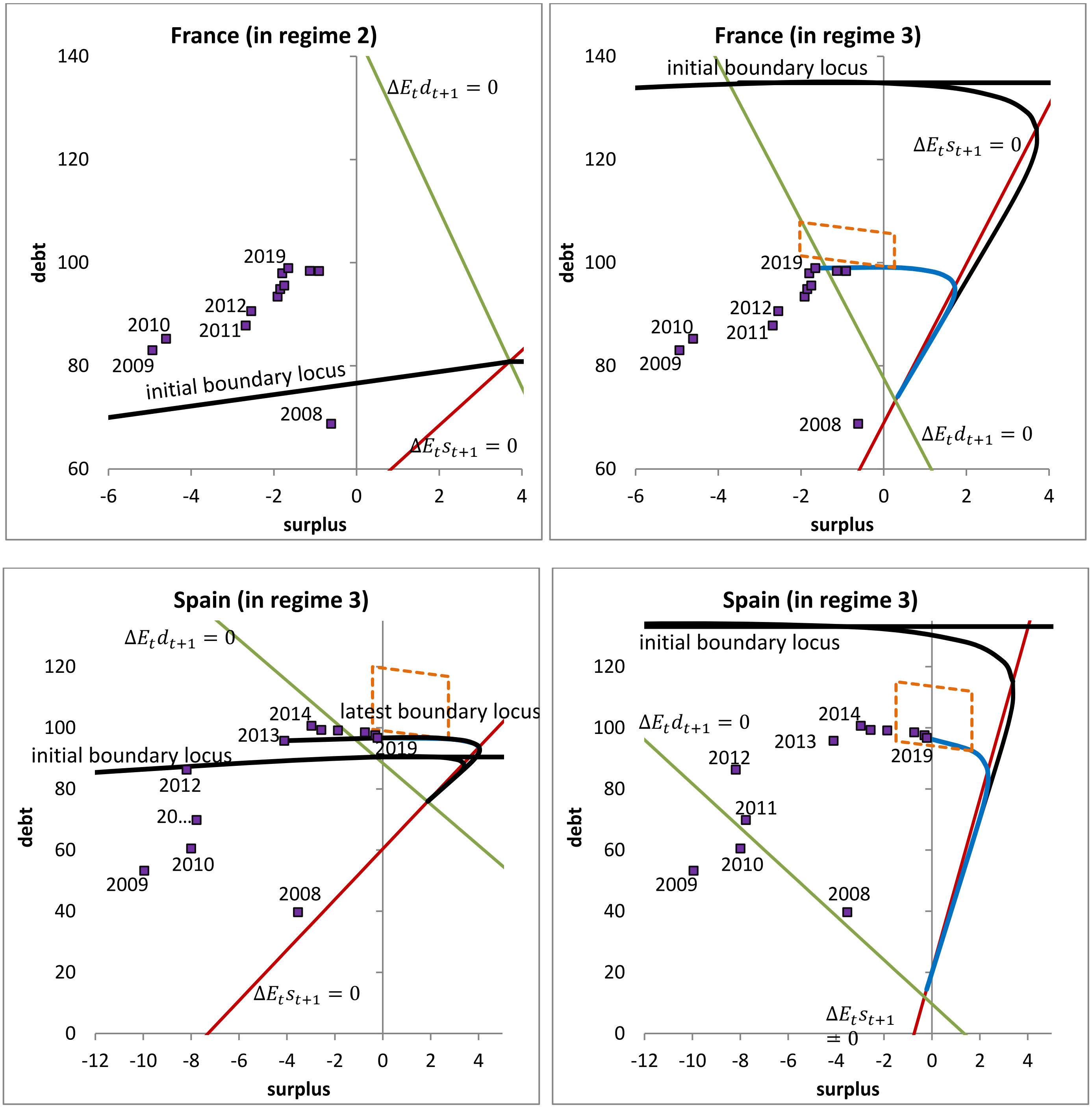


Figure 6: Italy and Japan