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Abstract

Economists often need to forecast temporally aggregated data, such as monthly or quarterly averages. However, when the underlying data is persistent, constructing forecasts with aggregated data is inefficient. We propose a new forecasting method, Period-End-Point Sampling (PEPS), which uses end-of-period data to create point-in-time forecasts for aggregated data. We show that PEPS forecasts rival the accuracy of bottom-up forecasts and substantially outperform forecasts constructed with averaged data. Importantly, the PEPS method allows models to maintain the lower frequency of the forecast target. Real-time forecast applications to monthly nominal 10-year bond yields and the real prices of gasoline and copper find that disaggregated forecasts can outperform the end-of-month no-change forecasts.

JEL classification: C1, C53, E47, F37, Q47
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1 Introduction

Economic series are often temporally aggregated, that is, averages or sums of higher-frequency data, such as monthly or quarterly averages. Forecasts of such series, especially when expressed in real terms, play a key role in expectation formation, central bank projections, and investment decisions.[1] Unfortunately, temporal aggregation introduces a loss of information contained in higher-frequency data, making forecasts constructed with temporally aggregated data inefficient (Zellner and Montmarquette, 1971; Tiao, 1972; Amemiya and Wu, 1972; Wei, 1978; Kohn, 1982; Lütkepohl, 1986). The main approach to avoid this information loss is the bottom-up (BU) approach, which consists of computing forecasts for the underlying high-frequency data and averaging the forecasts ex-post (Zellner and Montmarquette, 1971; Lütkepohl, 1986).[2] However, in practice, forecast and projection models are commonly implemented with averaged data due to the challenges of altering the frequency of the forecast model.

In this paper, we propose a general method for forecasting temporally aggregated data with disaggregated observations that allows the forecast model to maintain the same frequency as the target variable. This method, which we call Period-End-Point Sampling (PEPS), consists of constructing forecasts with point-sampled end-of-period observations and using these point forecasts as the forecast of the period average. For example, if the target series is a monthly average of daily data, PEPS estimates a monthly forecasting model with end-of-month observations. The resulting point forecasts for the end-of-month observations are then used as forecasts of the monthly average.

We show that like the BU approach, the PEPS approach improves on forecasts constructed with average data when the underlying data is persistent, as is often the case for economic series. The forecast efficiency of PEPS rivals that of the BU approach. A major advantage of PEPS is that it allows forecasters to maintain all variables within the same lower frequency as the forecasted series. This is often desirable in structural and multivariate models, which include other variables that are observed only at a lower frequency. PEPS is straightforward to implement as, in the simplest case, it only requires substituting aggregated observations for end-of-period observations when constructing forecasts. As such, PEPS is a particularly undemanding way of incorporating

[1]For example, forecasts of average prices might be necessary to accurately predict total costs or revenues. More generally, average observations more closely reflect average economic conditions over a certain period and are common in forecasts of real effective exchange rates and quarterly energy prices (see, e.g., Christiano and Eichenbaum, 1987; Baumeister and Kilian, 2014).

[2]An alternative approach relying on information from higher frequency data is Mixed Data Sampling (MIDAS, see, e.g. Ghysels et al., 2007; Andreou et al., 2013). This approach has only been applied in a multivariate context. In contrast, our primary focus is on univariate autoregressive moving average processes, for which the bottom-up approach is the principle existing method.

high-frequency information from the disaggregated data.

Using simulation analysis, we find that PEPS performs similarly to the efficient BU approach. Both approaches substantially improve on the forecasts constructed with averaged data, with up to 50% improvements in the mean-squared forecast error (MSFE) at the one-step-ahead prediction. Similar gains are found for the directional accuracy of the forecast. This finding complements existing studies, which have focused on the information loss from temporal aggregation for mean-squared forecast precision (e.g. Zellner and Montmarquette, 1971; Tiao, 1972; Amemiya and Wu, 1972; Lütkepohl, 1986, 2006).

In our empirical application, we examine the real-time forecast performance of alternative forecasts for the real price of retail gasoline, the nominal yield on 10-year U.S. bonds, and the real price of copper. For all series, we find large and robust improvements in forecast accuracy from employing disaggregated approaches at short horizons. For the one-month-ahead forecast, the MSFEs of the disaggagregated approaches are up to 50% lower than for forecasts constructed with averaged data, which is unprecedented in existing forecast applications for these series. Confirming our simulation results, the PEPS forecasts perform similarly to the BU forecasts. We also show the practical advantages of PEPS for multivariate models and models estimated with backcasted data, which can result in additional forecast improvements.

These findings contribute to the literature comparing BU and aggregated approaches, which has focused on aggregation from monthly to quarterly or from quarterly to yearly data (see, e.g., Zellner and Montmarquette, 1971; Wei, 1978; Lütkepohl, 1986; Athanasopoulos et al., 2011). In our simulations and empirical applications, the loss in forecast accuracy resulting from daily to monthly aggregation is much greater than the loss resulting from monthly to quarterly aggregation. This occurs because daily-to-monthly aggregation moves from a state of no aggregation to a substantive aggregation (typically 21 business days). In contrast, monthly to quarterly aggregations represent additional aggregations of already aggregated series. As such, using information from daily data results in forecast gains that are much larger than implied by the existing literature.

Our results complement a related body of existing work on the effect of aggregation on forecasts and the dynamics of economic series (see, e.g., Working, 1960; Brewer, 1973; Weiss, 1984; Rossana and Seater, 1995; Marcellino, 1999). Theoretical results for ARIMA models show that forecasts constructed with aggregated time series are less efficient than forecasts constructed with the BU approach (Tiao, 1972; Amemiya and Wu, 1972; Wei, 1978; Lütkepohl, 1986). This holds even when accounting for the effect of temporal aggregation on the ARIMA structure (Kohn, 1982) and under parameter uncertainty (Lütkepohl, 1986). We extend these results by showing that suitably constructed point forecasts can correct for the information loss from temporal aggregation and can be as efficient as the BU approach in forecasting averaged series.

2 The Method of Period-End-Point Sampling

2.1 Point Sampling and Temporal Aggregation

Consider a daily time series $y_{t,i}$ such that $t = 1, 2, \ldots, T$ indicates a lower-frequency period and $i = 1, 2, \ldots, n$ is the day within the period so that the last daily observation of period t is $y_{t,n}$. The first observation of the next month is $y_{t+1,1}$. The period-t average is given by

$$\bar{y}_t = \frac{1}{n}\sum_{i=1}^{n} y_{t,i}. \tag{1}$$

Throughout the paper, the forecaster's objective is to use information available at the end of period T to predict the k-periods-ahead average observation, $\bar{y}_{T+k}$.

An alternative way to reduce the frequency of daily observations is point sampling, which could, for example, consist of recording observations from a specific day i of each month. This form of sampling is also known as *skip sampling, systematic sampling, point-in-time sampling,* or *selective sampling* in the literature. A point-sampled series particularly relevant for this paper is the series of end-of-period observations, $y_{1,n}, y_{2,n}, \ldots, y_{T,n}$.

The form of sampling plays a subtle but critical role in determining the stochastic properties of the resulting series. In the previous literature, this role has been investigated separately for point sampling and temporal aggregation. In this paper, we exploit the advantages of selective sampling while maintaining the goal of forecasting aggregated variables.

The first advantage is that relative to averaging, point sampling tends to be less distortionary for the original data-generating process. For (V)AR processes, averaging always introduces an additional MA-component into the process, while this is not necessarily the case for selective sampling (Brewer, 1973; Weiss, 1984; Wei, 1981; Marcellino, 1999). For example, when the underlying series is generated by a random walk, averaging produces a moving average process (Working, 1960), while point sampling maintains the random-walk properties at the lower frequency.

The second advantage, which accrues to point sampling of end-of-period observations, is that the last observed instantaneous (non-averaged) observation can reflect the full information available to economic agents at the time the forecast is formed. This property is particularly relevant if the underlying series has a forward-looking component, such as prices of storable goods and assets, as famously argued by Fama (1970). Averaging dilutes the latest information contained in the last observed observation by averaging over past observations containing stale information. As a result, computations of forecasts with averaged data will generally result in a deterioration in forecast accuracy, which is not necessarily the case for forecasts computed with point-sampled data (Kohn, 1982).

2.2 Definition of PEPS

The key idea of this paper is to use a point forecast constructed with point-sampled end-of-period values as a forecast of the period average. We refer to this method as Period-End-Point Sampling (PEPS). Specifically, we use the observations $y_{1,n}, y_{2,n}, \ldots, y_{T,n}$ to construct a forecast of $y_{T+k,i}$ and then employ this forecast as a forecast of $\bar{y}_{T+k}$. Under the PEPS approach, model-based forecasts are thus estimated with end-of-period observations rather than period-average values.

More formally, let $\hat{\bar{y}}_{T+k}$, $k = 1, 2, \ldots, K$, be the forecast of $\bar{y}_{T+k}$ using PEPS. Let the model-implied conditional expectations be denoted by $\mathbf{E}_{T,n}[\cdot]$ to highlight the granularity of the information set. PEPS constructs a point-in-time forecast for day i in forecast period $T + k$ and uses that forecast as the forecast of the period average in $T + k$,

$$\hat{\bar{y}}_{T+k} = \mathbf{E}_{T,n}\left[y_{T+k,i}\right] \text{ , for any } i \text{ of period } T + k.$$

The most straightforward approach in practice is to construct a forecast of the end-of-period and then use the end-of-period forecast as the forecast of the period average:

$$\hat{\bar{y}}_{T+k} = \mathbf{E}_{T,n}\left[y_{T+k,n}\right] \text{ , } \forall\ T + k.$$

2.3 Intuition for PEPS: The AR(1) Case

The easiest way to provide intuition for the properties of the PEPS approach is to consider the case of the AR(1) model. Suppose that the daily series is generated by a stationary autoregressive model[3] with one lag,

$$y_{t,i} = \rho y_{t,i-1} + \epsilon_{t,i}\,, \qquad \text{for } i = 1,2,...,n;\ t = 1,2,...T. \tag{2}$$

Here, n is the number of daily observations within a period, and $\epsilon_{t,i} \sim iid(0, \sigma_\epsilon)$ denotes the daily innovation.

The goal is to construct a forecast of a temporally aggregated series, $\bar{y}_t$, which is obtained by averaging across $n \geq 2$ non-overlapping observations. Forecasts are constructed using the entire information set up to period T, including daily observations. We assume that ρ is known, in which case the bottom-up approach provides efficient forecasts and can be used as a reference point.

In this setting, the forecast for the observation on day i in period $T + k$ is given by

$$\mathbf{E}_{T,n}\left[y_{T+k,i}\right] = \rho^{(k-1)n+i} y_{T,n}\,. \tag{3}$$

Under the BU approach, the period-average forecast is given by the simple average of the daily forecasts. For example, for the one-month-ahead forecast ($k = 1$)

$$\mathbf{E}_{T,n}\left[\bar{y}_{T+1}\right] = \frac{1}{n}\sum_{i=1}^{n} \rho^i y_{T,n}. \tag{4}$$

2.3.1 Relationship between Point and Period-Average Forecasts

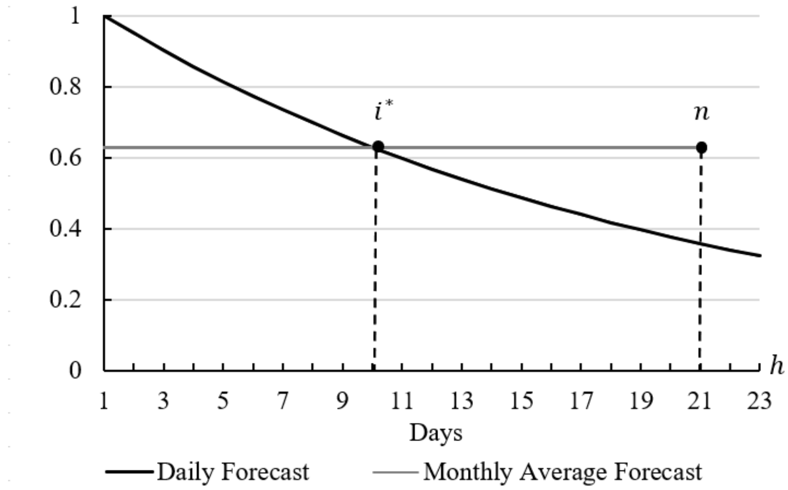
PEPS relies on using point forecasts for forecasts of period-average observations. This approach works better the closer these forecasts align. To provide intuition for the general relationship between point- and period-average forecasts, Figure 1 shows the monthly average ($n = 21$) and daily forecasts for an AR(1) model with $\rho = 0.95$ and $y_{T,n} = 1$. The monthly average forecast is constructed using the BU approach, and thus is a simple average over the daily forecasts.

Since the daily forecast is monotonically declining, it must be that the monthly average forecast intercepts the daily forecast on some day i^*, with $1 < i^* < n$. In practice, i^* can be found by solving

$$\rho^{(k-1)n+i^*} y_{T,n} = \frac{1}{n}\sum_{i=1}^{n} \rho^{(k-1)n+i} y_{T,n}, \tag{5}$$

[3] Kohn (1982) study stationary and nonstationary time series. However, Kohn explicitly states: "we look at the nonstationary case, and we assume that stationarity is achieved by differencing." For this reason, the theorems are for stationary data. This implicit assumption is also in Lütkepohl (1984, 1986, 2006) and Wei (1978).

Figure 1. Intersection of Point and Average Forecast



Note: Monthly forecast and monthly average for an AR(1) model with $\rho = 0.95$. Assumes $n = 21$ days in a month. The point forecast equals the period average forecast at i^*.

which yields

$$i^* = \frac{\ln\left[\frac{\rho^{(k-1)n+1}(\rho^n - 1)}{n(\rho-1)}\right]}{\ln(\rho)} - (k-1)n. \tag{6}$$

This implies that when ρ is known, we can determine a point forecast within a period that will be equal to the period-average forecast. Figure 2 graphs the values of i^* for alternative values of ρ for arbitrary $n \geq 2$ for the one-period ahead forecast. It indicates that $1 < i^* < (n-1)/2$. That is, for low values of ρ, forecasts for a point within the beginning of the period will be closest to the period average forecasts, while for values of ρ close to one, values closer to the middle of the month coincide with period average forecasts.

2.3.2 Relative Performance of End-of-Period Forecasts

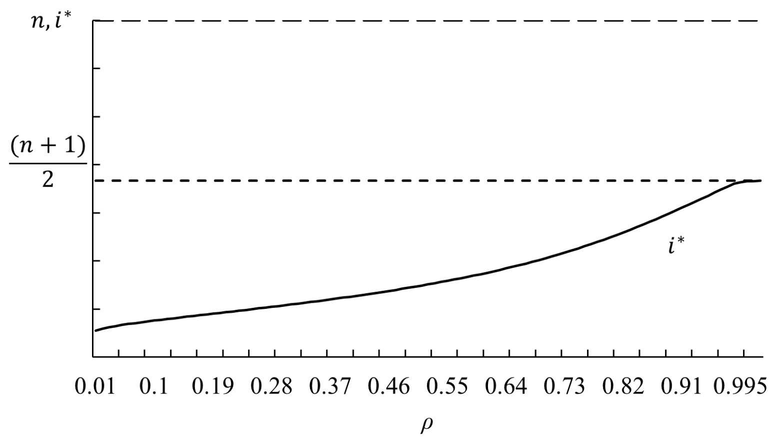
This section examines the properties of the PEPS forecasts when the end-of-period forecast is used as the forecast of the period average.

Claim 1. *Under the BU approach, the forecast error for the temporally aggregated AR(1) data with $n \geq 2$ and forecast horizon k is given by*

$$\sum_{i=1}^{n} \left[y_{T+k,i} - \mathbf{E}_{T,n}\left(y_{T+k,i}\right)\right] = \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=1}^{k} \rho^{(k-l)n-j+i}\varepsilon_{T+l,j}\,.$$

Proof. See appendix A1.1 □

Figure 2. Day i^* for Which the Average and Point Forecasts Intercept



Note: Forecast of point i^* for which the point forecast is equal to the period average forecast. Calculations for a daily AR(1) process with $0 < \rho < 1$.

Claim 2. *Using the end-of-period forecast as the forecast of temporally aggregated AR(1) data, for* $n \geq 2$ *and forecast horizon* k*, the forecast error is given by*

$$\sum_{i=1}^{n} \left[y_{T+k,i} - \mathbf{E}_{T,n} \left(y_{T+k,n} \right) \right] = \sum_{i=1}^{n} y_{T,n} \left(\rho^{(k-1)n+i} - \rho^{kn} \right) + \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{l=1}^{k} \rho^{(k-l)n-j+i} \varepsilon_{T+l,j} \, .$$

Proof. See appendix A1.2 □

Theorem 1. *For* $n \geq 2$*, the PEPS forecast of the end-of-period observation converges to the bottom-up forecast of temporally aggregated AR(1) data as*

(a) k *becomes large and as*

(b) $\rho \to 1$*.*

Proof. The result is immediate from Claim 1 and Claim 2 as the difference in the two forecasts is given by the first term on the right-hand side of Claim 2. □

Theorem (a) indicates that the difference between point and average forecasts is primarily an issue for short-horizon forecasts, as the forecasts converge at longer horizons. Moreover, Theorem (b) argues that independent of the forecast horizon, end-of-period point forecasts converge to efficient forecasts as the persistence of the data increases. This is noteworthy because this is precisely the situation in which aggregation matters most, in the sense that forecasts constructed from models estimated with average data perform poorly relative to the BU approach (Amemiya and Wu, 1972;

Tiao, 1972). As such, PEPS is expected to be useful precisely when recursive forecasts constructed with aggregate data are not useful. Due to the high persistence often found in economic series, such as prices of commodities or assets, this case is also particularly relevant in practice.

3 Simulated Forecast Performance

We quantify the efficiency of the different forecast approaches using simulation analysis. For this purpose, we assume that the high-frequency data, $y_{t,i}$, is observed for each day i in period t, and follows an autoregressive process of order one:

$$y_{t,i} = \rho y_{t,i-1} + \epsilon_{t,i}, \qquad \text{for } i = 0, 1, 2, ..., n;\ t = 1, 2, ...T. \tag{7}$$

where n is the number of business days in t (which can be the week, month, or quarter), and $\epsilon_{t,i} \sim N(0, 1)$. The objective is to forecast the k-period-ahead average series, $\hat{\bar{y}}_{T+k}$, where $\bar{y}_t = n^{-1} \sum_{i=1}^{n} y_{t,i}$. As a baseline, we simulate 40 years worth of data in addition to burning the first 500 daily observations. We use the first 75 percent of the sample for estimation and the remaining 25 percent as the forecast evaluation sample. This setup reflects common applications for daily financial data, which are typically available since the early 1980s. The setup is also consistent with the applications to macroeconomic variables in section 4.

We assume that the model structure is known but allow for parameter uncertainty. The autoregressive parameter is reestimated at every period with an expanding window, and forecasts are computed out-of-sample. It is known from theory that under point sampling of end-of-period data, the sampled data remains an AR(1) process, whereas the monthly average data is transformed into an ARMA(1,1) (Weiss, 1984). Consequently, forecasts computed with averaged data rely on an ARMA(1,1) model to avoid model misspecification.

We report two common forecast criteria, the MSFE ratio and the success ratio for directional accuracy. Both criteria are expressed relative to the period-average no-change forecast. This no-change benchmark is commonly used in forecasting applications for aggregated data and is most suitable to highlight the gains from using disaggregated forecast approaches.

The MSFE ratio for the k-steps-ahead forecast, $MSFE_k^{ratio}$, is calculated as the ratio of the

MSFE of the model-based forecast to the MSFE of the period-average no-change forecast:

$$MSFE_k^{ratio} = \frac{\sum_{q=1}^{Q}(\bar{y}_{q+k} - \hat{\bar{y}}_{q+k|q})^2}{\sum_{q=1}^{Q}(\bar{y}_{q+k} - \bar{y}_q)^2}, \tag{8}$$

where $q = 1, 2, 3, \ldots, Q$ denotes all periods of the evaluation sample, $\hat{\bar{y}}_{q+k|q}$ is the conditional forecast for the k-step-ahead aggregated observation, $\bar{y}_{q+k}$.

The directional accuracy is assessed using the mean directional accuracy, also known as the success ratio. It describes the fraction of times the forecasting model can correctly predict the change in the direction of the series of interest:

$$SR_k = \frac{1}{Q}\sum_{q=1}^{Q} \mathbb{1}[sgn(\bar{y}_{q+k} - \bar{y}_q) = sgn(\hat{\bar{y}}_{q+k|q} - \bar{y}_q)], \tag{9}$$

where $\mathbb{1}[\cdot]$ is an indicator function with 1 if true and 0 otherwise, and $sgn(\cdot)$ is a sign function with

$$sgn(x) = \begin{cases} 1 & x > 0 \\ -1 & x \leq 0 \end{cases}. \tag{10}$$

Under this definition, the success ratio equals one half ($SR_k = 0.5$) when there is no directional accuracy, while success ratios greater than one half ($SR_k > 0.5$) indicate directional predictability. A separate contribution of this paper is to examine, for the first time, the directional accuracy of disaggregated approaches for forecasts of temporally aggregated series since previous studies only consider the MSFE criterion (Zellner and Montmarquette, 1971; Tiao, 1972; Amemiya and Wu, 1972; Lütkepohl, 1986, 2006).

3.1 Forecast Models

To quantify the accuracy of alternative forecasting techniques, we compare the accuracy of five alternative forecasts.

The first approach that we consider is the BU approach. In the current setting, it involves first estimating the AR(1) model at the daily frequency and then averaging the forecasts to the period average ex post. Specifically, we estimate the model using the daily data, $y_{t,i}$:

$$y_{t,i} \quad = \quad \hat{\rho} y_{t,i-1} + \hat{\epsilon}_{t,i}, \quad \forall \; t \leq T\,. \tag{11}$$

The estimated parameter $\hat{\rho}$ is used to construct recursive model-based forecasts of the daily data, $\hat{y}_{T,n+h|T}$, where $h \geq 1$ is the forecast horizons in days. Then, the daily forecasts, $\hat{y}_{T+k,i|T}$, $k \geq 1$, are averaged to the period average:

$$\hat{\bar{y}}_{T+k|T} = \frac{1}{n}\sum_{i=1}^{n} \hat{y}_{T+k,i|T}, \quad \forall\ k \geq 1. \tag{12}$$

The second approach is the aggregated approach. In the current setting, it involves estimating an ARMA(1,1) model with period-average data to construct forecasts of $\bar{y}_{T+k}$. Specifically, we estimate the model using the monthly average data, $\bar{y}_t$:

$$\bar{y}_t \;\; = \;\; \tilde{\rho}\bar{y}_{t-1} + \tilde{\epsilon}_t + \tilde{\alpha}\tilde{\epsilon}_{t-1}, \quad \forall\ t \leq T\,. \tag{13}$$

The estimated parameters $\tilde{\rho}$ and $\tilde{\alpha}$ are used to construct recursive model-based forecasts of the monthly average, $\tilde{\bar{y}}_{T+k|T}$.

We next examine applications of period-end-point sampling (PEPS). The model is estimated with end-of-period data, and then point forecasts are constructed and used as the forecasts of the period average. Specifically, the AR(1) model is estimated at the period-t frequency using a time series of end-of-period values:

$$y_{t,n} \;\; = \;\; \check{\rho}y_{t-1,n} + \check{\epsilon}_t, \quad \forall\ t \leq T\,. \tag{14}$$

The first application of PEPS considers the use of the end-of-period point forecast as the forecast of the period average. In this case, the estimated parameter $\check{\rho}$ is used to construct recursive model-based forecasts of the end-of-period data, $\check{y}_{T+k,n|T}$. Then, the end-of-period forecasts are used as the forecasts of the period average $\check{\bar{y}}_{T+k} = \check{y}_{T+k,n|T}$.

The second application of PEPS uses the point forecast which is expected to be equivalent to the period average forecast, denoted $\check{y}_{T+k,i^*|T}$, derived in section 2. Note that if the parameter value was known, point sampling implies that $\check{\rho} = \rho^n$ (Zellner and Montmarquette, 1971). We can thus approximate the day that the PEPS forecast is equal to the BU forecast using the estimated parameter $\check{y}_{T+k,i^*|T} = \check{\rho}^{(k-1)+i^*/n}\hat{y}_{T,n}$. Then, the i^* point forecasts are used as the forecasts of the period averages $\check{\bar{y}}_{T+k} = \check{y}_{T+k,i^*|T}$.

Finally, we also consider the no-change forecast of the daily data $\hat{\bar{y}}_{T+k} = y_{T,n}$. For the special case when the daily series follows a random walk, $\rho = 1$, the end-of-month no-change forecast outperforms the period-average no-change forecast for all n, h (Ellwanger and Snudden, 2023a).

3.2 Simulated Performance

The comparison of the alternative forecasts of monthly average observations for alternative values of ρ is provided in Table 1. Values of the MSFE ratio less than one indicate mean-squared forecast improvements relative to the monthly average no-change forecast. Values of the success ratio above 0.5 indicate improvements in directional accuracy above random chance.

Table 1. One-Month-Ahead Forecast Performance of Alternative Forecast Approaches

Method	Aggregate	Bottom-Up	PEPS	PEPS(i*)	No-Change
Data	Average	Daily	EoM	EoM	Daily
ρ			MSFE Ratio		
1.00	0.94	0.54	0.55	0.54	0.54
	(0.045)	(0.062)	(0.064)	(0.062)	(0.062)
0.995	0.92	0.54	0.56	0.54	0.56
	(0.043)	(0.062)	(0.068)	(0.062)	(0.066)
0.99	0.89	0.54	0.57	0.54	0.58
	(0.043)	(0.062)	(0.072)	(0.062)	(0.071)
0.98	0.85	0.54	0.60	0.54	0.63
	(0.041)	(0.062)	(0.077)	(0.062)	(0.081)
0.95	0.75	0.53	0.63	0.53	0.77
	(0.042)	(0.062)	(0.081)	(0.062)	(0.114)
0.90	0.65	0.53	0.63	0.53	1.04
	(0.045)	(0.058)	(0.075)	(0.063)	(0.173)
ρ			Success Ratio		
1.00	0.58	0.74	0.74	0.74	0.74
	(0.043)	(0.040)	(0.040)	(0.040)	(0.040)
0.995	0.60	0.74	0.73	0.74	0.73
	(0.045)	(0.038)	(0.040)	(0.039)	(0.040)
0.99	0.61	0.74	0.73	0.73	0.73
	(0.040)	(0.038)	(0.040)	(0.039)	(0.040)
0.98	0.63	0.74	0.72	0.73	0.72
	(0.037)	(0.038)	(0.041)	(0.038)	(0.041)
0.95	0.67	0.74	0.69	0.72	0.69
	(0.038)	(0.039)	(0.044)	(0.038)	(0.044)
0.90	0.70	0.74	0.65	0.72	0.65
	(0.035)	(0.036)	(0.044)	(0.036)	(0.044)

Note: Comparison of monthly forecasts for 500 simulations of an AR(1) model at the daily frequency when estimated with alternative methods (standard deviation of the ratios in brackets). All MSFE and success ratios are expressed relative to the monthly average no-change forecast. Values of the MSFE ratio less than one indicate improvements over the monthly average no-change forecast. Values of the success ratio greater than 0.5 indicate improvements over random chance. The simulations are generated for 21 observations (business days) in a month and 40 years of data. The first 500 days are discarded, and the estimation sample is fixed at 75 percent of the sample, with the remaining 25 percent left as the evaluation period. The last column presents the forecasts from the no-change forecast based on the end-of-month (EoM) observation.

The last column of Table 1 shows the relative performance of the no-change forecast constructed with end-of-month values. When the data is persistent, $\rho > 0.9$, the end-of-month no-change forecast is a more accurate forecast of the monthly average than the monthly average no-change forecast, consistent with Tiao (1972). For large values of ρ, the forecast gains from the end-of-month no-change forecast relative to the monthly average no-change forecast can be substantial, with MSFE reductions approaching 46 percent. In terms of directional accuracy, the end-of-month no-change forecast outperforms the monthly average no-change forecast even for $\rho = 0.9$, demonstrating that temporal aggregation results for the MSFE ratio do not directly translate into results for directional accuracy.

The forecasts constructed with aggregated data are shown in the first column of Table 1. These forecasts outperform the period-average no-change forecast for all values of $\rho \leq 1$. However, for $\rho > 0.95$, the forecasts constructed with aggregated data perform worse than the end-of-month no-change forecast. This example shows that comparisons with the period-average no-change forecast can lead to spurious predictability, and illustrates the importance of comparing forecasts of aggregated data to the random forecast of the daily data to evaluate the usefulness of the forecasts (see Ellwanger and Snudden, 2023a).

In contrast, the relative performance of the BU forecasts (column three of Table 1) does not depend as much on the autocorrelation of the underlying data. For all $\rho < 1$, the bottom-up approach outperforms the end-of-month no-change forecast, demonstrating that it is generally desirable to exploit the disaggregated data.

The accuracy of the PEPS(i^*) forecasts, shown in column five of Table 1, is very similar to that of the BU forecasts. Consistent with the intuition of section 2, they yield large improvements relative to forecasts constructed with monthly average data. The directional accuracy of the forecasts constructed with the PEPS approach that targets the end-of-month observation (column four) outperforms the forecasts computed with averaged data for all values of ρ considered. Additionally, the end-of-month forecasts using PEPS outperforms the end-of-month no-change forecast for MSFE precision for all $\rho \leq 0.99$.

The exercises demonstrate that substantial gains in MSFE and directional accuracy can be obtained by using information from the underlying daily data for forecasts of the monthly average data. These results also suggest that point forecasts can provide effective forecasts of monthly average data.

3.3 Longer Horizons

The loss in relative forecast accuracy for forecasts computed with temporally aggregated data is generally largest at the one-step-ahead prediction, and decreases for longer forecast horizons (Amemiya and Wu, 1972; Tiao, 1972). The intuition for this result is that the aggregation bias is relatively fixed, whereas the forecast error resulting from unpredictable innovations increases with the forecast horizon. For the same reason, the relative performances of the BU forecasts, PEPS forecasts, and forecasts computed with aggregated data converge at longer horizons.

Table 2. Performance of Forecasts at Alternative Horizons

Model			ρ=0.995					ρ=0.95		
Method	Aggregate	Bottom-up	PEPS	PEPS(i*)	No-Change	Aggregate	Bottom-up	PEPS	PEPS(i*)	No-Change
Data	Average	Daily	EoM	EoM	Daily	Average	Daily	EoM	EoM	Daily
Horizon					MSFE Ratio					
1	0.92	0.54	0.56	0.54	0.56	0.75	0.53	0.63	0.53	0.77
	(0.043)	(0.062)	(0.068)	(0.062)	(0.066)	(0.042)	(0.062)	(0.081)	(0.062)	(0.114)
3	0.87	0.80	0.81	0.81	0.92	0.54	0.54	0.54	0.54	1.17
	(0.061)	(0.063)	(0.075)	(0.075)	(0.052)	(0.056)	(0.055)	(0.058)	(0.058)	(0.105)
6	0.77	0.75	0.76	0.76	0.98	0.50	0.50	0.50	0.50	1.20
	(0.100)	(0.098)	(0.107)	(0.107)	(0.040)	(0.061)	(0.062)	(0.062)	(0.062)	(0.107)
12	0.65	0.65	0.65	0.65	1.01	0.50	0.50	0.50	0.50	1.20
	(0.151)	(0.152)	(0.156)	(0.156)	(0.035)	(0.066)	(0.066)	(0.066)	(0.066)	(0.106)
					Success Ratio					
1	0.60	0.74	0.74	0.74	0.73	0.67	0.74	0.72	0.72	0.69
	(0.045)	(0.038)	(0.039)	(0.039)	(0.040)	(0.038)	(0.039)	(0.038)	(0.038)	(0.044)
3	0.63	0.65	0.65	0.65	0.60	0.74	0.74	0.74	0.74	0.55
	(0.047)	(0.043)	(0.043)	(0.043)	(0.041)	(0.038)	(0.037)	(0.038)	(0.038)	(0.043)
6	0.67	0.67	0.67	0.67	0.56	0.75	0.75	0.75	0.75	0.54
	(0.057)	(0.052)	(0.053)	(0.053)	(0.040)	(0.038)	(0.038)	(0.038)	(0.038)	(0.043)
12	0.71	0.71	0.71	0.71	0.53	0.75	0.75	0.75	0.75	0.54
	(0.066)	(0.064)	(0.064)	(0.064)	(0.042)	(0.041)	(0.041)	(0.041)	(0.041)	(0.042)

Note: Comparison of monthly forecasts at alternative forecast horizons for 500 simulations of an AR(1) model at the daily frequency when estimated with alternative methods (standard deviation of the ratios in brackets). All MSFE and success ratios are expressed relative to the monthly average no-change forecast. Values of the MSFE ratio less than one indicate improvements over the monthly average no-change forecast. Values of the success ratio greater than 0.5 indicate improvements over random chance. The simulations are generated for 21 observations (business days) in a month and 40 years of data. The first 500 days are discarded, and the estimation sample is fixed at 75 percent of the sample, with the remaining 25 percent left as the evaluation period. The last column presents the forecasts from the no-change forecast based on the end-of-month (EoM) observation.

This pattern is confirmed in Table 2, which shows the performance of the five forecasts (equations 11–14) at the 1-, 3-, 6-, and 12-month horizon. For the 3-month horizon and beyond, the performance of all model-based forecasts is similar for the case of $\rho = 0.95$. The performance of PEPS is very close to the efficient BU forecast beyond the one-month-ahead horizon. In contrast, the use of aggregated data results in the largest forecast errors at medium-run horizons when the daily data is persistent ($\rho = 0.995$). These results show that the relative performance of the BU and PEPS approaches convergence quickly, reinforcing the idea that the loss in forecast accuracy from using aggregated data occurs primarily at the one-step-ahead forecast.

3.4 Sampling Frequency

Table 3. Performance of Forecasts at Alternative Sampling Frequencies

Aggregation			Weekly					Quarterly		
Method	Aggregate	Bottom-Up	PEPS	PEPS(i*)	No-Change	Aggregate	Bottom-Up	PEPS	PEPS(i*)	No-Change
Data	Average	Daily	EoM	EoM	Daily	Average	Daily	EoM	EoM	Daily
ρ					MSFE Ratio					
1.00	0.94	0.65	0.65	0.65	0.65	0.95	0.52	0.54	0.52	0.52
	(0.021)	(0.030)	(0.029)	(0.029)	(0.030)	(0.086)	(0.104)	(0.115)	(0.104)	(0.101)
0.995	0.94	0.65	0.65	0.65	0.65	0.87	0.52	0.57	0.52	0.58
	(0.020)	(0.030)	(0.031)	(0.030)	(0.031)	(0.081)	(0.105)	(0.132)	(0.105)	(0.125)
0.99	0.93	0.64	0.65	0.64	0.66	0.82	0.52	0.60	0.52	0.65
	(0.019)	(0.030)	(0.032)	(0.030)	(0.031)	(0.081)	(0.105)	(0.142)	(0.106)	(0.151)
0.98	0.92	0.64	0.65	0.64	0.67	0.74	0.52	0.64	0.52	0.80
	(0.019)	(0.031)	(0.033)	(0.031)	(0.032)	(0.078)	(0.103)	(0.142)	(0.105)	(0.212)
0.95	0.88	0.63	0.66	0.63	0.70	0.62	0.52	0.63	0.54	1.29
	(0.019)	(0.030)	(0.035)	(0.030)	(0.035)	(0.082)	(0.098)	(0.128)	(0.175)	(0.411)
0.90	0.83	0.62	0.66	0.62	0.76	0.73	0.52	0.59	0.69	2.23
	(0.019)	(0.028)	(0.037)	(0.029)	(0.040)	(3.099)	(0.091)	(0.117)	(0.266)	(0.803)
ρ					Success Ratio					
1.00	0.57	0.70	0.70	0.70	0.70	0.58	0.75	0.74	0.75	0.75
	(0.022)	(0.019)	(0.019)	(0.019)	(0.019)	(0.075)	(0.068)	(0.070)	(0.067)	(0.068)
0.995	0.58	0.70	0.70	0.70	0.70	0.62	0.75	0.73	0.75	0.73
	(0.021)	(0.019)	(0.019)	(0.019)	(0.019)	(0.068)	(0.067)	(0.068)	(0.067)	(0.067)
0.99	0.58	0.70	0.70	0.70	0.70	0.65	0.75	0.73	0.75	0.72
	(0.019)	(0.019)	(0.019)	(0.019)	(0.020)	(0.064)	(0.068)	(0.070)	(0.067)	(0.068)
0.98	0.59	0.71	0.70	0.70	0.69	0.68	0.75	0.72	0.75	0.69
	(0.021)	(0.019)	(0.019)	(0.019)	(0.021)	(0.064)	(0.068)	(0.068)	(0.067)	(0.069)
0.95	0.61	0.71	0.70	0.71	0.69	0.72	0.75	0.72	0.67	0.65
	(0.022)	(0.018)	(0.018)	(0.018)	(0.020)	(0.061)	(0.064)	(0.061)	(0.119)	(0.078)
0.90	0.64	0.71	0.70	0.71	0.67	0.74	0.76	0.74	0.61	0.62
	(0.020)	(0.020)	(0.020)	(0.020)	(0.022)	(0.063)	(0.063)	(0.062)	(0.112)	(0.079)

Note: Comparison of one-period ahead weekly and quarterly forecasts for 500 simulations of an AR(1) model at the daily frequency when estimated with alternative methods (standard deviation of the ratios in brackets). All MSFE and success ratios are expressed relative to the monthly average no-change forecast. Values of the MSFE ratio less than one indicate improvements over the monthly average no-change forecast. Values of the success ratio greater than 0.5 indicate improvements over random chance. The simulations are generated for 5 and 63 observations (business days) in a week and quarter, respectively, and 40 years of data. The first 500 days are discarded, and the estimation sample is fixed at 75 percent of the sample, with the remaining 25 percent left as the evaluation period. The last column presents the forecasts from the no-change forecast based on the end-of-month (EoM) observation.

We now examine the relative performance at alternative frequencies of temporal aggregation. Table 3 reports the one-period ahead relative forecast gains for weekly and quarterly aggregation frequencies and values of ρ. For all $\rho \geq 0.90$, there are gains from using PEPS rather than averaged data at the weekly frequency. The gains from disaggregated methods are even greater at the quarterly frequency. This shows that PEPS is useful for aggregation frequencies commonly encountered in practice.

4 Application to Real-time Forecasts

4.1 Data

Our empirical applications consider real-time monthly average forecasts of three macroeconomic series: the nominal yield on 10-year U.S. government bonds, the real price of copper, and the real price of U.S. retail gasoline.

Data on U.S. retail gasoline prices are obtained from the U.S. Energy Information Administration (EIA). Daily closing spot prices of copper are obtained from the London Metal Exchange. Daily data for the yield on the 10-year Treasury bonds and the monthly index of U.S. industrial production are obtained from FRED. The price index is the seasonally adjusted U.S. consumer price index (CPI) from the real-time database of the Philadelphia Federal Reserve. The CPI is published with a one-month delay and is nowcasted using the average historical growth rate. A detailed description of the data sources and CPI nowcasting is provided in appendix A2.

Table 4. Descriptive Statistics

Real Series	Date Range	PACF(1)	*N*	Mean	Std. Dev.	Data Construction
			Monthly Average			
Copper	1986.4–2021.1	0.9908	421	2103.40	902.52	Monthly average
Backcasted Copper	1973.1–2021.1	0.9890	581	2159.44	930.95	Monthly average
Retail Gasoline	1991.2–2021.1	0.9789	363	1.01	0.30	Monthly average
10-year Treasury Yield	1973.1–2021.1	0.9956	581	6.15	3.22	Monthly average
			End of Month			
Copper	1986.4–2021.1	0.9908	421	2113.67	912.69	Last trading day
Backcasted Copper	1973.1–2021.1	0.9890	581	2165.63	936.92	Last trading day
Retail Gasoline	1991.1–2021.1	0.9789	364	1.01	0.30	Last Monday
10-year Treasury Yield	1973.1–2021.1	0.9944	581	6.15	3.24	Last trading day
			Daily			
Copper	1986.04.01–2021.01.31	0.9991	8837	2106.94	905.61	Closing prices
Retail Gasoline	1991.01.21–2021.01.25	0.9975	1579	1.01	0.30	Mondays
10-year Treasury Yield	1973.01.02–2021.01.29	0.9998	12075	6.16	3.22	Closing prices
			Monthly Index			
Consumer Price Inflation	1973.1–2021.1	0.9922	581	3.85	3.02	Monthly index
US Industrial Production	1973.1–2021.1	0.9637	581	2.47	3.87	Monthly index

Note: Copper and gasoline prices are reported in real terms, 10-year bond yields in nominal terms. Consumer price inflation and industrial production are reported in year-over-year growth rates in percent. The number of monthly or daily observations is denoted N. The 2021M4 vintages are used for the consumer price index. *PACF(1)* denotes the partial autocorrelation coefficient of the first lag.

The descriptive statistics for the end-of-month and monthly average series are reported in Table

4. Monthly averages are the simple average of daily closing prices. End-of-month observations are the closing price on the last trading day of each month. The exception is retail gasoline prices, which are collected by the EIA by weekly sampling of the price on Mondays. All high-frequency observations are available in real time and are not subject to historical revisions. Models are estimated using the available data, which starts in 1973M1 for yields, in 1986M4 for copper prices, and in 1991M2 for retail gasoline prices.

The real end-of-period observations, $R_{t,n}$, are constructed by applying the CPI index to the last observation of the period using $R_{t,n|t} = p_{t,n}/CPI_{t|t}$. This measure of end-of-month real prices is common for forecasts of end-of-month real prices, such as bilateral exchange rates (Meese and Rogoff, 1983) and primary commodities (West and Wong, 2014).

One advantage of using monthly data is that the monthly average and end-of-month prices can be backcasted using monthly average data. This is useful because the backcasting may provide a better estimate of the long-run mean, which could improve the forecast accuracy at longer horizons. The usefulness of backcasting is an empirical question and is quantified for the real price of copper. End-of-month and monthly average copper prices are backcasted to 1973M1 using the World Bank Monthly Average Commodities Price Data.[4]

For all of our series, the empirical variances of the end-of-period observations are very similar to the variances of the aggregated observations. The partial autocorrelations coefficient of the monthly real data indicates that all the series are highly persistent.

4.2 Forecast Techniques

All forecasts are computed out-of-sample using real-time methods. Specifically, we use historical vintages of (nowcasted) data available in the month of the forecast, and the models are re-estimated at every monthly step with an expanding window. The forecast evaluation period is 2000M1-2021M1.

Augmented Dickey-Fuller tests are used to test for unit roots and determine the appropriateness of estimating the models in levels or differences. The prices of gasoline and copper are estimated in log levels, and the interest rate in levels. The results suggest that the backcasted monthly real prices of copper are stationary in log real levels, but the nominal daily data and the non-backcasted monthly data are only stationary in differences. Moreover, both the daily and monthly gasoline prices and bond yields are stationary in differences. Accordingly, the models for nominal interest

[4] The Pink Sheet. https://www.worldbank.org/en/research/commodity-markets

rate are estimated in differences, and the models for the real backcasted copper are estimated in log levels, and real and non-backcasted copper and gasoline are estimated in growth rates.

For models estimated in monthly averages in real log levels, $\bar{r}_t = \ln(\bar{R}_t)$, the ARMA$(p, q)$ with p autoregressive parameters, q moving-average coefficients, and estimated innovations $\tilde{\epsilon}_t$ is given by:

$$\tilde{a}(L)\bar{r}_t \quad = \quad \tilde{c} + \tilde{b}(L)\tilde{\epsilon}_t\,, \qquad \forall\; t \leq T\,, \tag{15}$$

where $\tilde{c}$ is the estimated constant and L is the lag operator such that $Ly_t = y_{t-1}$, $\tilde{b}(L) = (1 + \tilde{\alpha}_1 L + \cdots + \tilde{\alpha}_q L^q)$, and $\tilde{a}(L) = (1 - \tilde{\rho}_1 L - \cdots - \tilde{\rho}_p L^p)$. These estimated parameters are used to construct recursive model-based forecasts, $\tilde{\bar{r}}_{T+k|T}$, which are converted back into real prices in levels, $\tilde{\bar{R}}_{T+k|T} = \exp\left(\tilde{\bar{r}}_{T+k|T}\right)$.

Forecasts of the monthly average nominal yields, $\bar{S}_t$, are estimated in differences $\bar{d}_t = \bar{S}_t - \bar{S}_{t-1}$. Then, the level forecasts are constructed using the model-implied difference, such that

$$\tilde{\bar{S}}_{T+k|T} = \tilde{\bar{d}}_{T+k|T} + \tilde{\bar{S}}_{T+k-1|T},\; \forall\; k \geq 1\,.$$

Similarly, forecasts constructed using monthly average real growth rates, $\bar{g}_t = \bar{R}_t/\bar{R}_{t-1} - 1$, are converted back into real-level forecasts using the model-implied net growth rate

$$\tilde{\bar{R}}_{T+k|T} = (1 + \tilde{\bar{g}}_{T+k|T}) \cdot \tilde{\bar{R}}_{T+k-1|T}\; \forall\; k \geq 1\,.$$

The first disaggregated approach that we consider is the BU approach. One potential complication with constructing BU forecasts for monthly average real data is that the CPI is only available at the monthly frequency, which may help explain why this approach has been overlooked in applications to real macroeconomic variables that are aggregated from a daily frequency (see for example, Box et al., 2015). However, as suggested by Benmoussa et al. (2023), this issue can be overcome by applying CPI forecasts to forecasts of nominal monthly average series.

Since all nominal daily prices are difference stationary, we estimate the model using the growth rate of nominal daily prices, $g_{t,i} = S_{t,i}/S_{t,i-1}$. The ARMA$(p, q)$ model is given by

$$\hat{a}(L)g_{t,i} \quad = \quad \hat{c} + \hat{b}(L)\hat{\epsilon}_{t,i}, \qquad \forall\; i,\; t \leq T. \tag{16}$$

The estimated parameters are used to construct recursive model-based forecasts of the growth rate of daily nominal prices, $\hat{g}_{T,n+h|T}$, where $h \geq 1$ is the forecast horizons in days. The forecasts for the level of the nominal price on day i of month $T+k$, given month T information, are based on the model-implied net growth rate:

$$\hat{S}_{T,n+h|T} = (1 + \hat{g}_{T,n+h|T}) \cdot \hat{S}_{T,n+h-1|T}\,, \quad \forall\ h \geq 1. \tag{17}$$

Then, nominal daily forecasts are averaged to the monthly frequency and converted into forecasts of real prices by deflating the monthly nominal forecasts by the expected CPI deflator:

$$\hat{\bar{R}}_{T+k|T} = \frac{n^{-1}\sum_{i=1}^{n}\hat{S}_{T+k,i|T}}{\mathbf{E}_{T,n}[CPI_{T+k|T}]}, \quad \forall\ k \geq 1. \tag{18}$$

The expected CPI, $C\hat{P}I_{T+k|T}$, is constructed by expanding the (nowcasted) current CPI observations with the average historical rate of inflation.

Yield forecasts are constructed similarly, except that the nominal daily data is transformed into differences, $d_{t,i} = S_{t,i} - S_{t,i-1}$, instead of growth rates. In this case, the forecasts of the series in levels are constructed by summing over the forecasted differences:

$$\hat{S}_{T,n+h|T} = \hat{d}_{T,n+h|T} + \hat{S}_{T,n+h-1|T}, \quad \forall\ h \geq 1. \tag{19}$$

The daily forecasts of the nominal series in levels are then averaged to the monthly frequency:

$$\hat{\bar{S}}_{T+k|T} = \frac{1}{n}\sum_{i=1}^{n}\hat{S}_{T+k,i|T}, \quad \forall\ k \geq 1. \tag{20}$$

The second disaggregated approach is PEPS. Implementing the PEPS approach is straightforward, as it merely requires replacing the time series of monthly average prices with the time series of real end-of-period observations, $R_{t,n}$, during the model estimation. Specifically, the ARMA(p,q) model is estimated at the monthly frequency with time series of end-of-month prices in log-real levels and expressed as:

$$\breve{a}(L)r_{t,n} \quad = \quad \breve{c} + \breve{b}(L)\breve{\epsilon}_t, \qquad \forall\ t \leq T. \tag{21}$$

The estimate is used to construct recursive model-based forecasts of the end-of-month values

$\check{r}_{T+k,n|T}$, which are converted back into real prices in levels, $\check{R}_{T+k,n|T} = \exp\left(\check{r}_{T+k,n|T}\right)$. Then the end-of-month forecasts are used as forecasts of the corresponding monthly average value, $\check{\bar{R}}_{T+k|T} = \check{R}_{T+k,n|T}$. PEPS forecasts using end-of-month nominal differences are converted into nominal levels following the approach outlined above.

From section 2, we know that for persistent processes, the monthly average forecast may be close to a point forecast for the middle of the month. In addition to the PEPS forecast for the end-of-month observation, we also report estimates for PEPS(i^*), which is obtained from a simple linear interpolation of the end-of-month forecasts of two adjacent months:

$$\check{y}_{T+k,i^*|T} = \omega \check{y}_{T+k,n|T} + (1-\omega)\check{y}_{T+k-1,n|T}, \quad \forall\ k \geq 1\,, \tag{22}$$

where ω is the forecast averaging weights. A parametric assessment of the daily forecasts, similar to Figure 1, suggests that for copper and bond yields, the end-of-month point forecast is a very close approximation of the period average. In contrast, for retail gasoline, an equal-weighted forecast, $\omega = 0.5$, is a close approximate to the period average. For heuristic purposes, we report the equal weighed forecasts for all series.

Finally, we also examine an application of PEPS in a two-variable VAR model for gasoline and U.S. industrial production estimated at the monthly frequency. This extension demonstrates the usefulness of PEPS in a setting where the monthly frequency needs to remain. The VAR(p) model with p autoregressive parameters can be expressed as:

$$(1 - \check{\mathbf{a}}(L))\mathbf{g}_t \quad = \quad \check{\mathbf{e}}_t, \quad \forall\ t < T\,, \tag{23}$$

where $\mathbf{g}_t$, is a 2x1 vector of growth rates, $\check{\mathbf{a}}(L)$ is the autoregressive parameter matrix of order p, and $\check{\mathbf{e}}_t$ is a 2x1 vector of innovations. The estimated parameters are used to construct recursive model-based forecasts of the growth rates, $\check{\bar{g}}_{T+k|T}$ which are converted back into real prices in levels in the same way as described above for the ARIMA forecasts.

It is not possible to apply the BU approach to this VAR as industrial production is observed at the monthly frequency. However, applying PEPS to a VAR model only involves replacing the monthly average growth rates, $\bar{g}_t$, in the vector $\mathbf{g}_t$, with the end-of-month growth rates $g_{t,n}$. Again, the estimated parameters are used to construct recursive forecasts of the growth rates, which are converted back into levels following the approach described for the ARIMA model.

Consistent with Rossana and Seater (1995), we employ information criteria for model selection.

Specifically, we employ the Akaike Information Criterion (AIC) to select autoregressive terms. The decision to omit moving average terms has negligible effects on the forecast performance of models and avoids convergence issues in the real-time estimations. Our results are quantitatively robust to alternative model selection criteria, as discussed in section 4.5.

4.3 Evaluation

For forecast evaluation, we report the MSFE ratio and the success ratio expressed relative to the end-of-month no-change forecast. The end-of-period no-change forecast reflects the null hypothesis that *all* future prices – both daily and period averages — are conditionally unpredictable, and thus corresponds to the traditional random walk forecast used in economics and finance (Ellwanger and Snudden, 2023a).

Tests against the end-of-month no-change forecast are implemented as follows. The MSFE ratio for the k-steps-ahead forecast, $MSFE_k^{ratio}$, is calculated as the ratio of the MSFE of the model-based forecast to the MSFE of the end-of-month no-change forecast:

$$MSFE_k^{ratio} = \frac{\sum_{q=1}^{Q}(\bar{R}_{q+k} - \hat{\bar{R}}_{q+k|q})^2}{\sum_{q=1}^{Q}(\bar{R}_{q+k} - R_{q,n|q})^2}, \tag{24}$$

where $q = 1, 2, \ldots, Q$ denotes all periods of the evaluation sample, $\hat{\bar{R}}_{q+k|q}$ is the conditional, model-implied real-time forecast for the k-month-ahead observation, $\bar{R}_{q+k}$, and $R_{q,n}$ is the time q end-of-period observation.

The null hypothesis of an equal MSFE for the model-based forecast relative to the no-change forecasts is tested following Diebold and Mariano (1995) and compared against standard normal critical values. P-values for tests relative to the monthly average are reported in parentheses.[5]

Directional accuracy is assessed via the success ratio, SR_k, indicating the fraction of times the forecasting model correctly predicts the change in direction of the series of interest:

$$SR_k = \frac{1}{Q}\sum_{q=1}^{Q} \mathbb{1}[sgn(\bar{R}_{q+k} - R_{q,n}) = sgn(\hat{\bar{R}}_{q+k|q} - R_{q,n|q})], \tag{25}$$

The null hypothesis that the success ratio is 0.5 (corresponding to the case that the directional prediction is completely random) is tested following Pesaran and Timmermann (2009), with the

[5]Under nested models, real-time data that is subject to revisions, and estimation uncertainty, the assumptions underlying Diebold and Mariano (1995) are not met (Diebold, 2015). As is standard, the tests are still reported with this caveat in mind.

corresponding p-values reported in parentheses.

A separate contribution of this paper is to test the predictability of the monthly average data for the three variables against the end-of-month no-change forecast. Previous forecasts of real retail gasoline prices and interest rates have only been compared against the monthly average no-change forecast or alternative models (see, e.g. Anderson et al., 2013; Baumeister et al., 2017; Johannsen and Mertens, 2021; Carriero et al., 2021). However, as demonstrated by Ellwanger and Snudden (2023a), forecasts of averaged data need to be tested against the no-change forecast based on non-averaged data to test the null hypothesis that the series is generated by a random walk.

4.4 Results

4.4.1 Real Price of Copper

The forecasts of the monthly average real price of copper are reported in Table 5. The forecasts constructed with monthly average data perform very poorly at short horizons, especially in terms of MSFE. While the forecasts constructed with backcasted monthly average data also perform poorly at short horizons, employing the backcasted copper prices results in substantially better long-horizon forecasts, which outperform the end-of-month no-change forecast in terms of directional accuracy at the two-year horizon. These results illustrate the advantages of using backcasted monthly data.

The third column reports the forecasts constructed using the BU approach. The forecasts improve upon the average forecasts in terms of MSFE precision at short horizons, but not in terms of directional accuracy. Moreover, the forecasts fail to outperform the end-of-month no-change forecast at any horizon and are especially poor at longer horizons. In fact, at horizons beyond six months, the BU approach is worse than the forecasts computed with backcasted average data. Although the BU approach is more efficient at short horizons, the inability to backcast daily data suggests that the approach fails to provide effective estimates of the long-run mean. This highlights a relative disadvantage of the BU approach in practice.

The end-of-month PEPS approach has slightly worse accuracy than the BU approach at short horizons when the data is not backcasted. However, when the end-of-month data is backcasted, the end-of-month PEPS forecasts perform better than the other approaches at almost all horizons. Moreover, the PEPS(i^*) forecasts, reported in the last column, have the lowest one-month-ahead MSFE ratios of all approaches and good directional accuracy at all horizons. The results illustrate

Table 5. Real-Time Model-Based Forecasts of the Real Price of Copper

Method	Aggregate	Aggregate	Bottom-up	PEPS	PEPS	PEPS(i*)
Data	Average	Ave Backcast	Daily	EoM Backcast	EoM	EoM Backcast
Horizon				MSFE Ratio		
1	1.76 (0.998)	1.70 (0.999)	**0.99 (0.299)**	**0.97 (0.376)**	1.05 (0.731)	**0.93 (0.096)**
3	1.14 (0.960)	1.11 (0.809)	1.03 (0.872)	1.00 (0.495)	1.06 (0.755)	**0.96 (0.337)**
6	1.20 (0.980)	1.09 (0.708)	1.06 (0.906)	1.05 (0.619)	1.18 (0.968)	1.01 (0.535)
12	1.26 (0.990)	1.07 (0.662)	1.14 (0.958)	1.06 (0.647)	1.29 (0.985)	1.04 (0.595)
24	1.47 (1.000)	1.09 (0.743)	1.29 (0.998)	1.07 (0.708)	1.55 (1.000)	1.06 (0.665)
				Success Ratio		
1	**0.51 (0.352)**	**0.55 (0.057)**	**0.54 (0.081)**	**0.52 (0.309)**	**0.52 (0.191)**	**0.52 (0.309)**
3	0.48 (0.759)	**0.52 (0.175)**	0.50 (0.726)	**0.54 (0.092)**	**0.54 (0.241)**	**0.56 (0.053)**
6	0.45 (0.980)	**0.54 (0.121)**	0.50 (0.720)	**0.52 (0.237)**	0.47 (0.923)	**0.52 (0.210)**
12	0.44 (0.893)	**0.58 (0.111)**	0.42 (0.977)	**0.58 (0.101)**	0.41 (1.000)	**0.58 (0.101)**
24	0.38 (1.000)	**0.62 (0.040)**	0.38 (1.000)	**0.61 (0.047)**	0.41 (1.000)	**0.61 (0.047)**

Note: Real-time, out-of-sample forecasts of the monthly average real price of copper in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the monthly average, respectively. AR(12) selected using AIC. Forecast criteria expressed relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio. "Backcast" data refers to backcasting monthly series using monthly average data to 1973M1.

the benefits of PEPS, which include increased accuracy for short horizons by avoiding temporal aggregation, while also allowing for improved accuracy at longer horizons from backcasting.

Table 5 also provides novel insights by testing for the first time whether of forecasts for real copper prices outperform the end-of-month no-change forecasts. The results show that the improvements upon this benchmark are only significant for long horizons, suggesting that the monthly average real price of copper is similar to asset prices in this regard.

4.4.2 Nominal Yields on 10-year Treasury Bonds

Table 6 presents the performance of forecasts of the monthly average nominal yields on 10-year U.S. government bonds. It also shows a loss in forecast accuracy at short horizons from using aggregated data, with an MSFE precision that is 81 percent worse than the end-of-month no-change forecast at the one-month-ahead prediction. However, at longer horizons, the forecasts based on aggregated data outperform the end-of-month no-change forecast. These improvements are significant at the 5 percent level for the MSFE at the two-year horizon, and for directional accuracy at the six-month horizon.

Relative to the forecasts computed with aggregated data, the BU approach improves the MSFE accuracy but fares worse in terms of directional accuracy at short horizons. Moreover, the BU

Table 6. Real-Time Model-Based Forecasts of the Nominal 10-Year Treasury Bonds

Method	Aggregate	Bottom-up	PEPS	PEPS(i*)
Data	Average	Daily	EoM	EoM
Horizon	MSFE Ratio			
1	1.81 (1.000)	1.02 (0.926)	1.11 (0.996)	1.02 (0.784)
3	1.06 (0.927)	1.02 (0.926)	1.01 (0.579)	1.00 (0.427)
6	1.03 (0.743)	1.01 (0.666)	**0.99 (0.433)**	**0.98 (0.269)**
12	**0.98 (0.337)**	**0.97 (0.231)**	**0.92 (0.090)**	**0.92 (0.097)**
24	**0.84 (0.006)**	**0.96 (0.198)**	**0.81 (0.004)**	**0.82 (0.004)**
	Success Ratio			
1	0.50 (0.460)	0.46 (0.767)	0.49 (0.444)	0.49 (0.444)
3	0.49 (0.524)	0.47 (0.848)	**0.57 (0.025)**	**0.56 (0.031)**
6	**0.57 (0.027)**	**0.51 (0.880)**	**0.58 (0.048)**	**0.56 (0.121)**
12	**0.54 (0.431)**	**0.62 (0.932)**	**0.59 (0.221)**	**0.56 (0.398)**
24	**0.58 (0.161)**	**0.57 (1.000)**	**0.59 (0.214)**	**0.59 (0.209)**

Note: Real-time, out-of-sample forecasts of the nominal monthly average 10-year Treasury bonds in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the average, respectively. AR(12) selected using AIC. Forecast criteria expressed relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio.

approach does not show significant gains in terms of MSFE or directional accuracy beyond the 6-month horizon, reinforcing our previous findings on the disadvantages of the BU approach at longer forecast horizons.

At the one-month-ahead horizon, the PEPS(i^*) forecast performs similarly to the BU forecasts in terms of the MSFE. Moreover, both PEPS approaches outperform the BU approach and the random walk forecast at longer horizons, which further corroborates the usefulness of PEPS for both short- and long-horizon forecasts.

4.4.3 Real Price of Retail Gasoline

Our last application examines forecasts of the real price of U.S. retail gasoline, see Table 7. This example differs from previous applications because the underlying data is point sampled weekly, such that the monthly averages are over fewer underlying observations, $n = 4$ instead of $n = 21$. Despite this difference, we observe substantial reductions in forecast accuracy from forecasts computed with aggregated data. Column 2 shows that the mean-squared accuracy from the model estimated with monthly average data is 72 percent worse than that of the end-of-month no-change forecast at the one-month horizon.

As expected, the BU and PEPS approaches perform much better at short horizons, and both

Table 7. Real-Time Model-Based Forecasts of the Real Price of Retail Gasoline

Method	Aggregate	Bottom-up	PEPS	PEPS(i*)
Data	Average	Daily	EoM	EoM
Horizon		MSFE Ratio		
1	1.72 (1.000)	**0.81 (0.024)**	**0.96 (0.312)**	**0.90 (0.010)**
3	1.01 (0.574)	**0.93 (0.003)**	**0.91 (0.048)**	**0.90 (0.011)**
6	**0.86 (0.059)**	**0.99 (0.132)**	**0.86 (0.032)**	**0.85 (0.018)**
12	1.08 (0.840)	1.06 (0.978)	1.09 (0.864)	1.08 (0.838)
24	1.25 (0.978)	1.14 (0.998)	1.27 (0.975)	1.26 (0.973)
		Success Ratio		
1	0.46 (0.929)	**0.62 (0.000)**	**0.55 (0.079)**	**0.55 (0.079)**
3	**0.51 (0.436)**	**0.54 (0.046)**	**0.55 (0.098)**	**0.55 (0.099)**
6	**0.58 (0.021)**	**0.52 (0.164)**	**0.61 (0.003)**	**0.61 (0.004)**
12	0.44 (0.979)	0.45 (0.908)	0.48 (0.921)	0.47 (0.944)
24	0.45 (0.943)	0.37 (0.995)	0.48 (0.912)	0.47 (0.947)

Note: Real-time, out-of-sample forecasts of the monthly average real price of retail gasoline in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the average, respectively. AR(8) selected using AIC. Forecast criteria expressed relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio.

approaches can significantly outperform the end-of-month no-change forecast in terms of the MSFE and directional accuracy. The BU forecasts perform better than the PEPS forecasts at the one-month horizon, but the PEPS forecasts perform better at the 6-month horizon.

Notably, the forecasting gains obtained from applying the BU and PEPS are much larger than the gains documented in previous studies, which have relied on monthly average prices (see, e.g., Anderson et al., 2013; Baumeister et al., 2017). Moreover, this is the first time that real price of gasoline forecasts have been shown to result in significant improvements relative to the end-of-month no-change forecast, indicating that the real price of gasoline is predictable even at short horizons.[6]

To further highlight the usefulness of the PEPS approach, we also explore the forecast accuracy from a two-variable VAR model for the real price of retail gasoline and U.S. industrial production. Since the industrial production series is only available monthly, the model is estimated at the monthly frequency. The results again show that the model-based forecast constructed using average data results in poor forecast performance relative to the end-of-month no-change forecast, Table 8. However, merely by replacing the monthly average data with the end-of-month data for

[6]This is non-trivial given that the end-of-month no-change forecast is over 50 percent more accurate in terms of MSFE and directional accuracy than the monthly average no-change forecast, see appendix A3.

Table 8. Real-Time VAR Forecasts of the Real Price of Retail Gasoline

Method	Aggregate	PEPS	PEPS(i*)
Data	Average	EoM	EoM
Horizon		MSFE Ratio	
1	1.71 (1.000)	**0.97 (0.357)**	**0.92 (0.032)**
3	1.09 (0.953)	**0.98 (0.352)**	**0.96 (0.165)**
6	1.03 (0.650)	1.03 (0.663)	1.01 (0.579)
12	1.12 (0.904)	1.15 (0.919)	1.13 (0.905)
24	1.26 (0.986)	1.32 (0.987)	1.30 (0.986)
		Success Ratio	
1	0.42 (0.998)	**0.58 (0.012)**	**0.58 (0.012)**
3	**0.55 (0.099)**	**0.58 (0.019)**	**0.57 (0.026)**
6	**0.55 (0.150)**	**0.58 (0.035)**	**0.60 (0.010)**
12	**0.50 (0.760)**	**0.53 (0.625)**	**0.53 (0.579)**
24	0.47 (0.965)	0.48 (1.000)	0.47 (1.000)

Note: Real-time, out-of-sample forecasts of the monthly average real price of retail gasoline in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the average, respectively. VAR(2) with industrial production and retail gasoline estimated in growth rates. Forecast criteria expressed relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio.

gasoline in the VAR, the forecast results improve substantially and outperform the end-of-month no-change forecast at short horizons. Moreover, the PEPS forecasts significantly improve in directional accuracy up to the six-month horizon. This example shows that it is easy to implement PEPS even in situations where altering the frequency of the model is not viable, which is often the case for central bank projection models or other low-frequency macroeconomic frameworks. By replacing the monthly average observations with the end-of-month observations, a forecaster can substantially improve the forecast accuracy of period-average observations.

4.5 Robustness

We find that any averaging of the end-of-month observations, such as two-day or weekly averages, systemically reduces accuracy of both the no-change forecasts and model-based forecasts that are estimated with the PEPS approach, see appendix A3. This pattern supports the idea that the last available information contained in the closing price reflects all available information about future levels.

The results presented in this paper are remarkably robust to alternative modeling choices. For example, similar results are obtained when model lags are selected using the Schwarz (1978) information criterion, as done by Rossana and Seater (1995). Moreover, the results remain qualitatively and, for the most part, quantitatively unchanged, when ARMA(1,1) models are used instead of the

AIC criterion, see appendix A3.3.

Model-based forecasts using disaggregated approaches are also superior for forecasts of quarterly average prices. This is consistent with the simulation evidence in section 3, showing that aggregation over more observations increases the information loss (see also Amemiya and Wu, 1972; Wei, 1978).

Finally, our main results also hold for alternative methods of nowcasting the CPI. Moreover, it can be shown that using ex-post revised data instead of real-time data does not affect any of our conclusions. This robustness is expected, as fluctuations in the CPI deflator are generally small compared to fluctuations in nominal yields and prices and tend to have minimal impact on forecasts (Baumeister and Kilian, 2012; Ellwanger and Snudden, 2023b).

5 Conclusion

We have proposed the method of period-end-price sampling (PEPS) to forecast persistent temporally aggregated data. It consists of constructing forecasts with point-sampled end-of-period observations and using these point forecasts as the forecast of the period average. We have shown that PEPS avoids the loss of information that is induced by aggregation and generally yields superior accuracy to forecasts computed from aggregated data. The gains are sizeable for short horizons and similar to those obtained from the traditional BU approach.

A practical advantage of PEPS is that it allows the forecast model to maintain the same frequency as the target variable. We have shown that this is a useful feature when high-frequency information needs to be combined with information from lower-frequency variables, for example, for multivariate models and backcasting series with low-frequency information. We have shown that it is straightforward to extend the PEPS approach to such cases and provide additional accuracy gains at both short and long forecast horizons. These results are particularly useful for forecasters who want to maintain models at a lower frequency, which is often the case for central bank projections and other macroeconomic forecasts.

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Online Appendix (Not intended for publication)

A1 Additional Proofs

A1.1 Proof of Claim 1

Proof. Under bottom-up, $\mathbf{E}_{T,n}(y_{T+k,i}) = \rho^{(k-1)n+i} y_{T,n}$. Suppose $k = 1$, then

$$\sum_{i=1}^{n} [y_{T+1,i} - \mathbf{E}_{T,n}(y_{T+1,i})] = y_{T+1,1} - \mathbf{E}_{T,n}(y_{T+1,1}) + y_{T+1,2} - \mathbf{E}_{T,n}(y_{T+1,2}) + \cdots + y_{T+1,n} - \mathbf{E}_{T,n}(y_{T+1,n}) \,. \tag{26}$$

Note that

$$\begin{aligned}
y_{T+1,1} - \mathbf{E}_{T,n}(y_{T+1,1}) &= \rho y_{T,n} + \varepsilon_{T+1,1} + \rho y_{T,n} = \varepsilon_{T+1,1} \\
y_{T+1,2} - \mathbf{E}_{T,n}(y_{T+1,2}) &= \rho y_{T+1,1} + \varepsilon_{T+1,2} + \rho^2 y_{T,n} = \rho\varepsilon_{T+1,1} + \varepsilon_{T+1,2} = \sum_{j=1}^{2} \rho^{2-i} \varepsilon_{T+1,i} \\
\vdots &= \vdots \\
y_{T+1,n} - \mathbf{E}_{T,n}(y_{T+1,n}) &= \sum_{j=1}^{n} \rho^{n-i} \varepsilon_{T+1,i} \,.
\end{aligned}$$

Therefore,

$$\sum_{i=1}^{n} [y_{T+1,i} - \mathbf{E}_{T,n}(y_{T+1,i})] = \varepsilon_{T+1,1} + \sum_{j=1}^{2} \rho^{2-i} \varepsilon_{T+1,i} + \cdots + \sum_{j=1}^{n} \rho^{n-i} \varepsilon_{T+1,i} = \sum_{i=1}^{n} \sum_{j=1}^{i} \rho^{i-j} \varepsilon_{T+1,j} \,.$$

Hence, the relation in Claim 1 holds. Assume the relation in Claim 1 holds for $k - 1$, then

$$\begin{aligned}
\sum_{i=1}^{n} \left[y_{T+(k-1),i} - \mathbf{E}_{T,n}\left(y_{T+(k-1),i}\right)\right] &= y_{T+(k-1),1} - \mathbf{E}_{T,n}\left(y_{T+(k-1),1}\right) + y_{T+(k-1),2} - \mathbf{E}_{T,n}\left(y_{T+(k-1),2}\right) \\
&\quad + \cdots + y_{T+(k-1),n} - \mathbf{E}_{T,n}\left(y_{T+(k-1),n}\right) \\
&= \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{l=1}^{(k-1)} \rho^{((k-1)-l)n-j+i} \varepsilon_{T+l,j}
\end{aligned} \tag{27}$$

In particular, we have that

$$y_{T+(k-1),n} - \mathbf{E}_{T,n}\left(y_{T+(k-1),n}\right) = \rho^{(k-1)n} y_{T,n} + \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j} \varepsilon_{T+l,j} - \rho^{(k-1)n} y_{T,n} \,. \tag{28}$$

We show now that the relation holds for any k. Consider

$$\begin{aligned}
y_{T+k,1} - \mathbf{E}_{T,n}\left(y_{T+k,1}\right) &= \rho y_{T+k-1,n} + \varepsilon_{T+k,1} - \rho^{(k-1)n+1} y_{T,n} \\
&= \rho \left[\rho^{(k-1)n} y_{T,n} + \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j} \varepsilon_{T+l,j} \right] + \varepsilon_{T+k,1} - \rho^{(k-1)n+1} y_{T,n} \\
&= \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+1} \varepsilon_{T+l,j} + \varepsilon_{T+k,1} \\
y_{T+k,2} - \mathbf{E}_{T,n}\left(y_{T+k,2}\right) &= \rho y_{T+k,1} + \varepsilon_{T+k,2} - \rho^{(k-1)n+2} y_{T,n} \\
&= \rho^{2} y_{T+k-1,n} + \rho \varepsilon_{T+k,1} + \varepsilon_{T+k,2} - \rho^{(k-1)n+2} y_{T,n} \\
&= \rho^{2} \left[\rho^{(k-1)n} y_{T,n} + \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j} \varepsilon_{T+l,j} \right] + \rho \varepsilon_{T+k,1} + \varepsilon_{T+k,2} - \rho^{(k-1)n+2} y_{T,n} \\
&= \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+2} \varepsilon_{T+l,j} + \sum_{j=1}^{2} \rho^{2-j} \varepsilon_{T+k,j} \\
\vdots \quad &= \quad \vdots \\
y_{T+k,n} - \mathbf{E}_{T,n}\left(y_{T+k,n}\right) &= \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+n} \varepsilon_{T+l,j} + \sum_{j=1}^{n} \rho^{n-j} \varepsilon_{T+k,j} \\
&= \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+n} \varepsilon_{T+l,j} + \sum_{j=1}^{n} \rho^{n-j} \varepsilon_{T+k,j} \\
&= \sum_{j=1}^{n} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+n} \varepsilon_{T+l,j} + \sum_{j=1}^{n} \rho^{n-j} \varepsilon_{T+k,j} \,,
\end{aligned}$$

using (28) in the first equality above. Hence, we have that

$$\begin{aligned}
\sum_{i=1}^{n} \left[y_{T+k,i} - \mathbf{E}_{T,n}\left(y_{T+k,i}\right) \right] = \sum_{i=1}^{n} \Bigg[& \sum_{j=1}^{i} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+1} \varepsilon_{T+l,j} + \varepsilon_{T+k,1} \\
& + \sum_{j=1}^{i} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+2} \varepsilon_{T+l,j} + \sum_{j=1}^{2} \rho^{2-j} \varepsilon_{T+k,j} \\
& + \cdots \\
& + \sum_{j=1}^{i} \sum_{l=1}^{k-1} \rho^{(k-l)n-j+n} \varepsilon_{T+l,j} + \sum_{j=1}^{n} \rho^{n-j} \varepsilon_{T+k,j} \Bigg] \\
= & \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{l=1}^{k} \rho^{(k-l)n-j+i} \varepsilon_{T+l,j} \,.
\end{aligned} \tag{29}$$

□

A1.2 Proof of Claim 2

Proof. First, for $k = 1$, we have

$$\sum_{i=1}^{n} [y_{T+1,i} - \mathbf{E}_{T,n}[y_{T+1,n}]] = y_{T+1,1} - \mathbf{E}_{T,n}[y_{T+1,n}] + y_{T+1,2} - \mathbf{E}_{T,n}[y_{T+1,n}] \\ + \cdots + y_{T+1,n} - \mathbf{E}_{T,n}[y_{T+1,n}]$$

where

$$\begin{aligned} y_{T+1,1} - \mathbf{E}_{T,n}[y_{T+1,n}] &= \rho y_{T,n} + \epsilon_{T+1,1} - \rho^n y_{T,n} = \rho y_{T,n}(1-\rho^{n-1}) + \epsilon_{T+1,1} \\ y_{T+1,2} - \mathbf{E}_{T,n}[y_{T+1,n}] &= \rho y_{T+1,1} + \epsilon_{T+1,2} - \rho^n y_{T,n} = \rho^2 y_{T,n} + \rho\epsilon_{T+1,1} + \epsilon_{T+1,2} - \rho^n y_{T,n} \\ &= \rho^2 y_{T,n}(1-\rho^{n-2}) + \sum_{i=1}^{2} \rho^{2-i}\epsilon_{T+1,i} \\ \vdots &= \vdots \\ y_{T+1,n} - \mathbf{E}_{T,n}[y_{T+1,n}] &= \sum_{i=1}^{n} \rho^{n-i}\epsilon_{T+1,i}\,. \end{aligned}$$

Therefore,

$$\begin{aligned} \sum_{i=1}^{n} [y_{T+1,i} - \mathbf{E}_{T,n}[y_{T+1,n}]] &= \rho y_{T,n}(1-\rho^{n-1}) + \epsilon_{T+1,1} \\ &+ \rho^2 y_{T,n}(1-\rho^{n-2}) + \sum_{i=1}^{2} \rho^{2-i}\epsilon_{T+1,i} \\ &+ \cdots \\ &+ \sum_{i=1}^{n} \rho^{n-i}\epsilon_{T+1,i} \\ &= \sum_{i=1}^{n-1} \rho^i y_{T,n}(1-\rho^{n-i}) + \sum_{i=1}^{n}\sum_{j=1}^{i} \rho^{i-j}\epsilon_{T+1,j}. \end{aligned}$$

The relation in Claim 2 holds. Assume it holds for $k-1$. As in Claim 1, we have that

$$y_{T+(k-1),n} - \mathbf{E}_{T,n}\left[y_{T+(k-1),n}\right] = y_{T,n}\left(\rho^{(k-1-1)n+n} - \rho^{(k-1)n}\right) + \sum_{j=1}^{n}\sum_{l=1}^{k-1}\rho^{(k-l)n-j}\varepsilon_{T+l,j} \tag{30}$$

We show now that the relation holds for any k. Consider

$$\begin{aligned}
y_{T+k,1} - \mathbf{E}_{T,n}\left[y_{T+k,n}\right] &= \rho y_{T+k-1,n} + \varepsilon_{T+k,1} - \rho^{kn} y_{T,n} \\
&= y_{T,n}\left(\rho^{(k-1-1)n+n+1} - \rho^{(k-1)n+1}\right) + \sum_{j=1}^{n}\sum_{l=1}^{k-1}\rho^{(k-l)n-j+1}\varepsilon_{T+l,j} + \varepsilon_{T+k,1} \\
y_{T+k,2} - \mathbf{E}_{T,n}\left[y_{T+k,n}\right] &= y_{T,n}\left(\rho^{(k-1-1)n+n+2} - \rho^{(k-1)n+2}\right) + \sum_{j=1}^{n}\sum_{l=1}^{k-1}\rho^{(k-l)n-j+2}\varepsilon_{T+l,j} + \sum_{j=1}^{2}\rho^{2-j}\varepsilon_{T+k,j} \\
\vdots &= \vdots \\
y_{T+1,n} - \mathbf{E}_{T,n}\left[y_{T+1,n}\right] &= y_{T,n}\left(\rho^{(k-1-1)n+n+n} - \rho^{(k-1)n+n}\right) + \sum_{j=1}^{n}\sum_{l=1}^{k-1}\rho^{(k-l)n-j+n}\varepsilon_{T+l,j} + \sum_{j=1}^{n}\rho^{n-j}\varepsilon_{T+k,j} \\
&= y_{T,n}\left(\rho^{(k-1)n+n} - \rho^{kn}\right) + \sum_{j=1}^{n}\sum_{l=1}^{k-1}\rho^{(k-l)n-j+n}\varepsilon_{T+l,j} + \sum_{j=1}^{n}\rho^{n-j}\varepsilon_{T+k,j}
\end{aligned}$$

using (30) in the first equality above. Therefore,

$$\sum_{i=1}^{n}\left[y_{T+k,i} - \mathbf{E}_{T,n}\left(y_{T+k,n}\right)\right] = \sum_{i=1}^{n} y_{T,n}\left(\rho^{(k-1)n+i} - \rho^{kn}\right) + \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{l=1}^{k}\rho^{(k-l)n-j+i}\varepsilon_{T+l,j}\,.$$

□

A2 Data Appendix

This appendix describes the construction of the data series used in the empirical exercises.

Copper spot prices Daily closing spot prices of grade A copper at the London Metal Exchange were obtained from Bloomberg (LMCADY). Monthly average data used for backcasting is obtained from the World Bank Pink sheets.

Interest rate Daily data on interest rates were obtained from Board of Governors of the Federal Reserve System. The long-term rate is the Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis (DGS10), retrieved from FRED, Federal Reserve Bank of St. Louis.

U.S. retail gasoline Nominal weekly gasoline prices are obtained from Table 14 of the EIA's Weekly Petroleum Status Report. The specific series is "Weekly U.S. Regular All Formulations Retail Gasoline Prices". The weekly price corresponds to the price on the Monday of the week and is published on the same day, except on government holidays, for which the data is released on Tuesday. The end-of-month observation is the last observed weekly price; i.e., the price on the last Monday of each month. The monthly price is a simple average over all weekly observations within a month, which is identical to the monthly series "Regular Motor Gasoline, All Areas, Retail Price" published in the Monthly Energy Review.

Industrial production The index of U.S. industrial production is obtained from the Board of Governors of the Federal Reserve System, Industrial Production: Total Index (IPB50001N), retrieved from FRED, Federal Reserve Bank of St. Louis.

Consumer price index Real-time vintages of the seasonally adjusted U.S. consumer price index are obtained from the real-time database of the Philadelphia Federal Reserve.

Nowcasts of CPI for real forecasts Missing real-time observations for the CPIs are nowcasted using the average historical growth rate from 1973M1. All vintages of U.S. CPI are available and are observed with a one-month publication delay.

A3 Robustness Results

A3.1 Comparison of No-Change Forecasts

Table A1. End-of-Month Versus Monthly Average No-change Forecasts

Freq.	Daily			Weekly	
Series	Simulated Ran. Walk	10 Year Bonds	Copper	Simulated Ran. Walk	Retail Gasoline
Horizon					
1	0.54	0.56 (0.000)	0.55 (0.000)	0.71	0.47 (0.000)
3	0.89	0.84 (0.000)	0.89 (0.007)	0.83	0.86 (0.002)
6	0.95	0.92 (0.000)	0.96 (0.027)	0.88	0.98 (0.131)
12	0.97	0.95 (0.000)	0.99 (0.321)	0.94	1.00 (0.491)
1	0.74	0.75 (0.000)	0.71 (0.000)	0.75	0.76 (0.000)
3	0.61	0.65 (0.000)	0.60 (0.000)	0.68	0.62 (0.000)
6	0.58	0.66 (0.000)	0.55 (0.020)	0.66	0.55 (0.008)
12	0.55	0.64 (0.000)	0.56 (0.004)	0.63	0.56 (0.013)

Note: Real-time, out-of-sample forecasts in levels, 1992M1–2021M1. End-of-month no-change forecasts relative to the monthly average no-change forecast, with p-values reported in parentheses.

A3.2 Alternative Averaging

Table A2. Alternative End-of-Month No-changes Versus Monthly Average No-change Forecasts

Freq.	Daily			Weekly	
Series	Simulated Ran. Walk	10 Year Bonds	Copper	Simulated Ran. Walk	Retail Gasoline
Average			MSFE Ratio		
EoM	0.54	0.56 (0.000)	0.55 (0.000)	0.71	0.47 (0.000)
2 days	0.55	0.58 (0.000)	0.56 (0.000)	-	-
1 week	0.62	0.65 (0.000)	0.62 (0.000)	-	-
2 weeks	0.74	0.76 (0.000)	0.73 (0.000)	0.75	0.62 (0.000)
			Success Ratio		
EoM	0.74	0.75 (0.000)	0.71 (0.000)	0.75	0.76 (0.000)
2 days	0.73	0.73 (0.000)	0.73 (0.000)	-	-
1 week	0.71	0.72 (0.000)	0.72 (0.000)	-	-
2 weeks	0.68	0.67 (0.000)	0.68 (0.000)	0.66	0.75 (0.000)

Note: Real-time, out-of-sample forecasts in levels, 1992M1–2021M1. End-of-month no-change forecasts relative to the monthly average no-change forecast, with p-values reported in parentheses.

A3.3 Alternative Parameterizations

The results of section 4 rely on the Akaike information criteria (Akaike, 1998) to select the degree of autoregressive parameters. Here, we show that these results are qualitatively similar when an ARMA(1,1) is used to construct the forecasts. Again, there are gains in short-horizons forecast accuracy by using the bottom-up and PEPS approaches compared to constructing forecasts with averaged data, see Tables A3 –A5. For the real price of copper, the use of fewer lags worsens the mid-horizon accuracy, but improves the accuracy at the longest horizons. For bond yields and the real price of gasoline, the disaggregated techniques show fewer instances of statistically significant forecasts at medium forecast horizons. These exercises indicate that the differences in forecast accuracy between aggregated and disaggregated approaches are not driven by a particular choice of the lag length.

Table A3. Real-Time ARMA(1,1) Forecasts of the Real Price of Copper

Method	Aggregate	Aggregate	Bottom-up	PEPS	PEPS	PEPS(i*)
Data	Average	Ave Backcast	Daily	EoM Backcast	EoM	EoM Backcast
Horizon				MSFE Ratio		
1	1.72 (0.999)	1.66 (1.000)	1.01 (0.919)	**0.96 (0.248)**	1.05 (0.910)	**0.96 (0.084)**
3	1.13 (0.997)	1.06 (0.862)	1.03 (0.896)	**0.96 (0.218)**	1.05 (0.902)	**0.96 (0.156)**
6	1.10 (0.985)	**0.98 (0.408)**	1.06 (0.908)	**0.94 (0.234)**	1.08 (0.914)	**0.94 (0.197)**
12	1.14 (0.975)	**0.93 (0.249)**	1.13 (0.958)	**0.92 (0.215)**	1.17 (0.959)	**0.91 (0.189)**
24	1.29 (0.999)	**0.85 (0.025)**	1.29 (0.998)	**0.84 (0.019)**	1.37 (0.997)	**0.84 (0.016)**
				Success Ratio		
1	**0.51 (0.362)**	**0.52 (0.249)**	0.47 (0.837)	0.47 (0.842)	0.44 (0.964)	0.47 (0.842)
3	0.48 (0.830)	**0.52 (0.213)**	0.49 (0.755)	**0.54 (0.100)**	0.49 (0.965)	**0.54 (0.141)**
6	0.45 (0.993)	**0.55 (0.089)**	0.49 (0.819)	**0.55 (0.072)**	0.49 (1.000)	**0.55 (0.083)**
12	0.46 (0.773)	**0.61 (0.031)**	0.42 (0.985)	**0.64 (0.006)**	0.46 (1.000)	**0.64 (0.008)**
24	0.41 (0.998)	**0.66 (0.004)**	0.38 (1.000)	**0.68 (0.001)**	0.48 (1.000)	**0.68 (0.001)**

Note: Real-time, out-of-sample forecasts of the monthly average real price of copper in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the average, respectively. Forecast criteria relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio. "Backcast" data refers to using monthly average data to backcast the sampled daily data to 1973M1.

Table A4. Real-Time ARMA(1,1) Forecasts of the Nominal 10-Year Treasury Bonds

Method	Aggregate	Bottom-up	PEPS	PEPS(i*)
Data	Average	Daily	EoM	EoM
Horizon		MSFE Ratio		
1	1.77 (1.000)	1.01 (0.886)	1.03 (0.840)	1.00 (0.546)
3	1.08 (0.986)	1.00 (0.716)	1.01 (0.861)	1.01 (0.854)
6	1.04 (0.958)	1.00 (0.427)	1.01 (0.610)	1.00 (0.619)
12	1.03 (0.872)	**0.98 (0.205)**	**0.98 (0.259)**	**0.98 (0.252)**
24	**0.98 (0.293)**	**0.96 (0.178)**	**0.96 (0.198)**	**0.96 (0.192)**
		Success Ratio		
1	0.48 (0.650)	0.44 (0.997)	**0.52 (0.115)**	**0.52 (0.115)**
3	**0.53 (0.082)**	0.49 (1.000)	0.48 (0.745)	0.47 (0.816)
6	0.50 (0.552)	**0.53 (1.000)**	**0.53 (0.608)**	**0.53 (0.577)**
12	**0.51 (0.841)**	**0.63 (1.000)**	**0.61 (0.984)**	**0.61 (0.984)**
24	0.50 (0.993)	**0.58 (1.000)**	**0.57 (1.000)**	**0.57 (1.000)**

Note: Real-time, out-of-sample forecasts of the nominal monthly average 10-year Treasury bonds in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the average, respectively. Forecast criteria relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio.

Table A5. Real-Time ARMA(1,1) Forecasts of the Real Price of Retail Gasoline

Method	Aggregate	Bottom-up	PEPS	PEPS(i*)
Data	Average	Daily	EoM	EoM
Horizon		MSFE Ratio		
1	1.76 (1.000)	**0.80 (0.020)**	**0.89 (0.026)**	**0.91 (0.001)**
3	1.15 (1.000)	**0.94 (0.101)**	1.01 (0.622)	1.00 (0.411)
6	1.1 (0.984)	1.05 (0.974)	1.06 (0.939)	1.05 (0.937)
12	1.15 (0.972)	1.14 (0.995)	1.14 (0.948)	1.13 (0.945)
24	1.28 (0.991)	1.19 (1.000)	1.28 (0.988)	1.26 (0.987)
		Success Ratio		
1	0.42 (0.994)	**0.63 (0.000)**	**0.56 (0.027)**	**0.56 (0.027)**
3	0.41 (1.000)	**0.54 (0.031)**	**0.52 (0.398)**	**0.52 (0.330)**
6	0.42 (1.000)	**0.51 (0.221)**	0.49 (0.974)	0.49 (0.956)
12	0.48 (0.970)	0.43 (0.938)	0.49 (0.942)	0.49 (0.975)
24	0.48 (0.949)	0.38 (0.998)	0.47 (0.950)	0.47 (0.950)

Note: Real-time, out-of-sample forecasts of the monthly average real price of retail gasoline in levels, 2000M1–2021M1. PEPS and PEPS(i^*) use end-of-month and middle-of-month forecasts as the forecast of the average, respectively. Forecast criteria relative to the end-of-month no-change forecast, with p-values reported in parentheses. Bold values indicate forecast gains relative to the no-change forecast for the MSFE ratio and gain in directional accuracy for the success ratio.